Induction 2009-12

Question Number	Scheme	Marks
4	When n = 1, LHS = $\frac{1}{1\times 2}=\frac{1}{2}$, RHS = $\frac{1}{1+1}=\frac{1}{2}$. So LHS = RHS and result true for n = 1	B1
	Assume true for n = k; $\sum_{r=1}^{k}\frac{1}{r(r+1)}=\frac{k}{k+1}$ and so $\sum_{r=1}^{k+1}\frac{1}{r(r+1)}=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}$	M1
	$\sum_{r=1}^{k+1}\frac{1}{r(r+1)}=\frac{k(k+2)+1}{(k+1)(k+2)}=\frac{k^2+2k+1}{(k+1)(k+2)}=\frac{(k+1)^2}{(k+1)(k+2)}=\frac{k+1}{k+2}$	M1 A1
	and so result is true for n = k + 1 (and by induction true for $n\in\mathbf{Z}^+$)	B1 **[5]**

Notes:

Evaluate both sides for first B1
Final two terms on second line for first M1
Attempt to find common denominator for second M1.
Second M1 dependent upon first.
$\frac{k+1}{k+2}$ for A1
'Assume true for $n=k$ 'and 'so result true for $n=k+1$' and correct solution for final B1

Question Number	Scheme	Marks
Q8 (a)	$f(1) = 5 + 8 + 3 = 16$, (which is divisible by 4). ($\therefore$ True for n = 1).	B1
	Using the formula to write down f(k + 1), $f(k+1) = 5^{k+1} + 8(k+1) + 3$	M1 A1
	$f(k+1) - f(k) = 5^{k+1} + 8(k+1) + 3 - 5^k - 8k - 3$ $= 5(5^k) + 8k + 8 + 3 - 5^k - 8k - 3 = 4(5^k) + 8$	M1 A1
	$f(k+1) = 4(5^k + 2) + f(k)$, **which is divisible by** 4	A1ft
	$\therefore$ True for $n = k + 1$ **if** true for $n = k$. True for n = 1, $\therefore$ true for all n.	A1cso (7)
(b)	For n = 1, $\begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^1$ **(**$\therefore$ True for n = 1.)	B1
	$\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2k+1 & -2k \\ 2k & 1-2k \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 2k+3 & -2k-2 \\ 2k+2 & -2k-1 \end{pmatrix}$	M1 A1 A1
	$= \begin{pmatrix} 2(k+1)+1 & -2(k+1) \\ 2(k+1) & 1-2(k+1) \end{pmatrix}$	M1 A1
	$\therefore$ True for $n = k + 1$ **if** true for $n = k$. **True for *n*** = 1, $\therefore$ **true for all *n***	A1 cso (7) **[14]**
(a) Alternative for 2nd M:	$f(k+1) = 5(5^k) + 8k + 8 + 3$ M1 $= 4(5^k) + 8 + (5^k + 8k + 3)$ A1 or $= 5(5^k + 8k + 3) - 32k - 4$ $= 4(5^k + 2) + f(k)$, or $= 5f(k) - 4(8k+1)$ which is divisible by 4 A1 (or similar methods)	
Notes Part (b) Alternative	(a) B1 Correct values of 16 or 4 for n = 1 or for n = 0 (Accept "is a multiple of") M1 Using the formula to write down f(k + 1) A1 Correct expression of f(k+1) (or for f(n +1) M1 Start method to connect f(k+1) with f(k) as shown A1 correct working toward multiples of 4, A1 ft result including f(k +1) as subject, A1cso conclusion (b) B1 correct statement for n = 1 or n = 0 First M1: Set up product of two appropriate matrices – product can be either way round A1 A0 for one or two slips in simplified result A1 A1 all correct simplified A0 A0 more than two slips M1: States in terms of (k + 1) A1 Correct statement A1 for induction conclusion May write $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2k+3 & -2k-2 \\ 2k+2 & -2k-1 \end{pmatrix}$**.** Then may or may not complete the proof. This can be awarded the second M (substituting k + 1)and following A (simplification) in part (b). The first three marks are awarded as before . Concluding that they have reached the same matrix and therefore a result will then be part of final A1 cso but also need other statements as in the first method.	

Question Number	Scheme	Marks
Q8	(a) $\sum_{r=1}^{1} r^3 = 1^3 = 1$ and $\frac{1}{4}\times 1^2 \times 2^2 = 1$	B1
	Assume true for $n = k$:	
	$\sum_{r=1}^{k+1} r^3 = \frac{1}{4}k^2(k+1)^2 + (k+1)^3$	B1
	$\frac{1}{4}(k+1)^2\left[k^2 + 4(k+1)\right] = \frac{1}{4}(k+1)^2(k+2)^2$	M1 A1
	$\therefore$ True for $n = k + 1$ if true for $n = k$. True for $n = 1$, $\therefore$ true for all n.	A1cso (5)
	(b) $\sum r^3 + 3\sum r + \sum 2 = \frac{1}{4}n^2(n+1)^2 + 3\left(\frac{1}{2}n(n+1)\right), + 2n$	B1, B1
	$= \frac{1}{4}n\left[n(n+1)^2 + 6(n+1) + 8\right]$	M1
	$= \frac{1}{4}n\left[n^3 + 2n^2 + 7n + 14\right] = \frac{1}{4}n(n+2)(n^2+7)$ (*)	A1 A1cso (5)
	(c) $\sum_{15}^{25} = \sum_{1}^{25} - \sum_{1}^{14}$ with attempt to sub in answer to part (b)	M1
	$= \frac{1}{4}(25\times 27\times 632) - \frac{1}{4}(14\times 16\times 203) = 106650 - 11368 = 95282$	A1 (2) **[12]**
	Notes (a) Correct method to identify $(k+1)^2$ as a factor award M1 $\frac{1}{4}(k+1)^2(k+2)^2$ award first A1 All three elements stated somewhere in the solution award final A1 (b) Attempt to factorise by n for M1 $\frac{1}{4}$ and $n^3 + 2n^2 + 7n + 14$ for first A1 (c) no working 0/2	

Question Number	Scheme	Marks
9.	(a) If $n=1$, $\sum_{r=1}^{n} r^2 = 1$ and $\frac{1}{6}n(n+1)(2n+1) = \frac{1}{6}\times 1\times 2\times 3 = 1$, so true for n = 1.	B1
	Assume result true for $n = k$	M1
	$\sum_{r=1}^{k+1} r^2 = \frac{1}{6}k(k+1)(2k+1) + (k+1)^2$	M1
	$= \frac{1}{6}(k+1)(2k^2+7k+6)$ or $= \frac{1}{6}(k+2)(2k^2+5k+3)$ or $= \frac{1}{6}(2k+3)(k^2+3k+2)$	A1
	$= \frac{1}{6}(k+1)(k+2)(2k+3)$ $= \frac{1}{6}(k+1)(\{k+1\}+1)(2\{k+1\}+1)$ or equivalent	dM1
	True for $n = k + 1$ if true for $n = k$, (and true for n = 1) so true by induction for all n.	A1cso (6)
	Alternative for (a) After first three marks B M M1 as earlier :	B1M1M1
	May state RHS $= \frac{1}{6}(k+1)(\{k+1\}+1)(2\{k+1\}+1) = \frac{1}{6}(k+1)(k+2)(2k+3)$ for third M1	dM1
	Expands to $\frac{1}{6}(k+1)(2k^2+7k+6)$ and show equal to $\sum_{r=1}^{k+1} r^2 = \frac{1}{6}k(k+1)(2k+1)+(k+1)^2$ for A1	A1
	So true for $n = k + 1$ if true for $n = k$, and true for n = 1, so true by induction for all n.	A1cso (6)
	(b) $\sum_{r=1}^{n}(r^2+5r+6) = \sum_{r=1}^{n} r^2 + 5\sum_{r=1}^{n} r + \quad (\sum_{r=1}^{n} 6)$	M1
	$\frac{1}{6}n(n+1)(2n+1) + \frac{5}{2}n(n+1), \quad +6n$	A1, B1
	$= \frac{1}{6}n[(n+1)(2n+1)+15(n+1)+36]$	M1
	$= \frac{1}{6}n[2n^2+18n+52] = \frac{1}{3}n(n^2+9n+26)$ or $a = 9$, $b = 26$	A1 (5)
	(c) $\sum_{r=n+1}^{2n}(r+2)(r+3) = \frac{1}{3}2n(4n^2+18n+26) - \frac{1}{3}n(n^2+9n+26)$	M1 A1ft
	$\frac{1}{3}n(8n^2+36n+52-n^2-9n-26) = \frac{1}{3}n(7n^2+27n+26)$ (*)	A1cso (3) **14 marks**
	Notes: (a) B1: Checks n = 1 on both sides and states true for n = 1 here or in conclusion M1: **Assumes true** for $n = k$ (should use one of these **two** words) M1: Adds (k+1)th term to sum of k terms A1: Correct work to support proof M1: Deduces $\frac{1}{6}n(n+1)(2n+1)$ with $n = k + 1$ A1: Makes induction statement. Statement true for n = 1 here could contribute to B1 mark earlier	

	Question 9 Notes continued: (b) M1: Expands and splits (but allow 6 rather than sigma 6 for this mark) A1: first two terms correct B1: for 6*n* M1: Take out factor *n*/6 or *n*/3 correctly – no errors factorising A1: for correct factorised cubic or for identifying *a* and *b* (c) M1: Try to use $\sum_{1}^{2n}(r+2)(r+3)-\sum_{1}^{n}(r+2)(r+3)$ with previous result used **at least once** A1ft Two correct expressions for their a and b values A1: Completely correct work to printed answer

Question Number	Scheme		Marks
9.	$u_{n+1} = 4u_n + 2$, $u_1 = 2$ and $u_n = \frac{2}{3}(4^n - 1)$		
	$n=1$; $u_1 = \frac{2}{3}(4^1 - 1) = \frac{2}{3}(3) = 2$ So u_n is true when $n=1$.	Check that $u_n = \frac{2}{3}(4^n - 1)$ yields 2 when $n=1$.	B1
	Assume that for $n=k$ that, $u_k = \frac{2}{3}(4^k - 1)$ is true for $k \in \mathbb{Z}^+$.		
	Then $u_{k+1} = 4u_k + 2$		
	$= 4\left(\frac{2}{3}(4^k - 1)\right) + 2$	Substituting $u_k = \frac{2}{3}(4^k - 1)$ into $u_{n+1} = 4u_n + 2$.	M1
	$= \frac{8}{3}(4)^k - \frac{8}{3} + 2$	An attempt to multiply out the brackets by 4 or $\frac{8}{3}$	M1
	$= \frac{2}{3}(4)(4)^k - \frac{2}{3}$		
	$= \frac{2}{3}4^{k+1} - \frac{2}{3}$		
	$= \frac{2}{3}(4^{k+1} - 1)$	$\frac{2}{3}(4^{k+1} - 1)$	A1
	Therefore, the general statement, $u_n = \frac{2}{3}(4^n - 1)$ is true when $n = k+1$. (As u_n is true for $n=1$,) then u_n is true for all positive integers by mathematical induction	Require 'True when n=1', 'Assume true when *n*=*k*' and 'True when $n = k+1$' then true for all *n* o.e.	A1
			(5) **[5]**

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Question Number	Scheme	Notes	Marks
9. (a)	$n=1;\quad \text{LHS} = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^1 = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$ $\text{RHS} = \begin{pmatrix} 3^1 & 0 \\ 3(3^1-1) & 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$ As LHS = RHS, the matrix result is true for $n = 1$.	Check to see that the result is true for $n = 1$.	B1
	Assume that the matrix equation is true for $n = k$, ie. $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^k = \begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}$		
	With $n = k+1$ the matrix equation becomes $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^k \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$		
	$= \begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix} \text{ or } \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}\begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}$	$\begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}$ by $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$	M1
	$= \begin{pmatrix} 3^{k+1}+0 & 0+0 \\ 9(3^k-1)+6 & 0+1 \end{pmatrix} \text{ or } \begin{pmatrix} 3^{k+1}+0 & 0+0 \\ 6.3^k+3(3^k-1) & 0+1 \end{pmatrix}$	Correct unsimplified matrix with no errors seen.	A1
	$= \begin{pmatrix} 3^{k+1} & 0 \\ 9(3^k)-3 & 1 \end{pmatrix}$		
	$= \begin{pmatrix} 3^{k+1} & 0 \\ 3\left(3(3^k)-1\right) & 1 \end{pmatrix}$		
	$= \begin{pmatrix} 3^{k+1} & 0 \\ 3(3^{k+1}-1) & 1 \end{pmatrix}$	Manipulates so that $k \to k+1$ on at least one term. Correct result with no errors seen with some working between this and the previous A1	dM1 A1
	If the result is true for $n = k$, (1) then it is now true for $n = k+1$. (2) As the result has shown to be true for $n = 1$, (3) then the result is true for all n. (4) **All 4 aspects need to be mentioned at some point for the last A1.**	Correct conclusion with all previous marks earned	A1 cso
			(6)

Question Number	Scheme	Notes	Marks
9. (b)	$f(1) = 7^{2-1} + 5 = 7 + 5 = 12$,	Shows that $f(1) = 12$.	B1
	{which is divisible by 12}. {$\therefore$ f(n) is divisible by 12 when $n = 1$.}		
	Assume that for $n = k$, $f(k) = 7^{2k-1} + 5$ is divisible by 12 for $k \in ¢^{+}$.		
	So, $f(k+1) = 7^{2(k+1)-1} + 5$	Correct unsimplified expression for $f(k+1)$.	B1
	giving, $f(k+1) = 7^{2k+1} + 5$		
	$\therefore f(k+1) - f(k) = \left(7^{2k+1} + 5\right) - \left(7^{2k-1} + 5\right)$	Applies $f(k+1) - f(k)$. No simplification is necessary and condone missing brackets.	M1
	$= 7^{2k+1} - 7^{2k-1}$		
	$= 7^{2k-1}\left(7^2 - 1\right)$	Attempting to isolate 7^{2k-1}	M1
	$= 48\left(7^{2k-1}\right)$	$48\left(7^{2k-1}\right)$	A1cso
	$\therefore f(k+1) = f(k) + 48\left(7^{2k-1}\right)$, which is divisible by 12 as both $f(k)$ and $48\left(7^{2k-1}\right)$ are both divisible by 12.(1) If the result is true for $n = k$, (2) then it is now true for $n = k+1$. (3) As the result has shown to be true for $n = 1$,(4) then the result is true for all *n*. (5). **All 5 aspects need to be mentioned at some point for the last A1.**	Correct conclusion with no incorrect work. Don't condone missing brackets.	A1 cso
	There are other ways of proving this by induction. See appendix for 3 alternatives. **If you are in any doubt consult your team leader and/or use the review system.**		(6)
			12

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Question Number	Scheme	Notes	Marks
Aliter **8.** (c) **Way 2**	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2\sqrt{3}\,x^{-\frac{1}{2}} = \frac{2\sqrt{3}}{\sqrt{3}} = 2$		B1
	Gives $y - 12 = 2(x - 3)$	Uses (3, 12) and their "2" to find the equation of the tangent.	M1
	$x = -12 \Rightarrow y - 12 = 2(-12 - 3)$	Substitutes their x from (a) into their tangent	M1
	$y = -18$		
	So the coordinates of X are $(-12, -18)$.		A1
			(4)

Question Number	Scheme	Notes	Marks
Aliter **9.** (b) **Way 2**	$\mathrm{f}(1) = 7^{2-1} + 5 = 7 + 5 = 12$,	Shows that $\mathrm{f}(1) = 12$.	B1
	{which is divisible by 12}. {$\therefore$ f (n) is divisible by 12 when $n = 1$.}		
	Assume that for $n = k$,		
	$\mathrm{f}(k) = 7^{2k-1} + 5$ is divisible by 12 for $k \in ¢^{+}$.		
	So, $\mathrm{f}(k+1) = 7^{2(k+1)-1} + 5$	Correct expression for $\mathrm{f}(k+1)$.	B1
	giving, $\mathrm{f}(k+1) = 7^{2k+1} + 5$		
	$7^{2k+1} + 5 = 49 \times 7^{2k-1} + 5$	Attempt to isolate 7^{2k-1}	M1
	$= 49 \times (7^{2k-1} + 5) - 240$	M1 Attempt to isolate $7^{2k-1} + 5$	M1
	$\mathrm{f}(k+1) = 49 \times \mathrm{f}(k) - 240$	Correct expression in terms of f(k)	A1
	As both f(k) and 240 are divisible by 12 then so is f(k + 1). If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all n.	Correct conclusion	A1
			(6)

Question Number	Scheme	Notes	Marks
Aliter			
9. (b)	$\text{f}(1) = 7^{2-1} + 5 = 7 + 5 = 12,$	Shows that $\text{f}(1) = 12$.	B1
Way 3	{which is divisible by 12}. $\{\therefore \text{f}(n)$ is divisible by 12 when $n = 1.\}$		
	Assume that for $n = k$, f(k) is divisible by 12		
	so $\text{f}(k) = 7^{2k-1} + 5 = 12m$		
	So, $\text{f}(k+1) = 7^{2(k+1)-1} + 5$	Correct expression for $\text{f}(k+1)$.	B1
	giving, $\text{f}(k+1) = 7^{2k+1} + 5$		
	$7^{2k+1} + 5 = 7^2.7^{2k-1} + 5 = 49 \times 7^{2k-1} + 5$	Attempt to isolate 7^{2k-1}	M1
	$= 49 \times (12m - 5) + 5$	Substitute for m	M1
	$\text{f}(k+1) = 49 \times 12m - 240$	Correct expression in terms of m	A1
	As both $49 \times 12m$ and 240 are divisible by 12 then so is f(k + 1). If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all n.	Correct conclusion	A1
			(6)

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Question Number	Scheme	Notes	Marks
Aliter **9.** (b) **Way 4**			
	$f(1) = 7^{2-1} + 5 = 7 + 5 = 12,$	Shows that $f(1) = 12$.	B1
	{which is divisible by 12}. {$\therefore$ f (n) is divisible by 12 when $n = 1$.}		
	Assume that for $n = k$, $f(k) = 7^{2k-1} + 5$ is divisible by 12 for $k \in ¢^{+}$.		
	$f(k+1) + 35f(k) = \underline{7^{2(k+1)-1} + 5} + 35(7^{2k-1} + 5)$	Correct expression for $f(k+1)$.	B1
	$f(k+1) + 35f(k) = 7^{2k+1} + 5 + 35(7^{2k-1} + 5)$	Add appropriate multiple of f(k) For 7^{2k} this is likely to be 35 (119, 203,.) For 7^{2k-1} 11 (23, 35, 47,..)	M1
	giving, $7.7^{2k} + 5 + 5.7^{2k} + 175$	Attempt to isolate 7^{2k}	M1
	$= 180 + 12 \times 7^{2k} = 12\left(15 + 7^{2k}\right)$	Correct expression	A1
	$\therefore f(k+1) = 12\left(7^{2k} + 15\right) - 35f(k)$. As both f($k$) and $12\left(7^{2k} + 15\right)$ are divisible by 12 then so is f(k + 1). If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all n.	Correct conclusion	A1
			(6)

Question Number	Scheme	Notes	Marks
6(a)	$n=1, \text{LHS}=1^3=1, \text{RHS}=\frac{1}{4}\times1^2\times2^2=1$	Shows **both** LHS = 1 **and** RHS = 1	B1
	Assume true for n = k		
	When n = k + 1 $\sum_{r=1}^{k+1} r^3 = \frac{1}{4}k^2(k+1)^2+(k+1)^3$	Adds $(k+1)^3$ to the given result	M1
	$=\frac{1}{4}(k+1)^2[k^2+4(k+1)]$	Attempt to factorise out $\frac{1}{4}(k+1)^2$	dM1
		Correct expression with $\frac{1}{4}(k+1)^2$ factorised out.	A1
	$=\frac{1}{4}(k+1)^2(k+2)^2$ Must see 4 things: <u>true for n = 1</u>, <u>assumption true for n = k</u>, <u>said true for n = k + 1</u> and therefore <u>true for all n</u>	Fully complete proof with no errors and comment. All the previous marks must have been scored.	A1cso
	See extra notes for alternative approaches		(5)
(b)	$\sum(r^3-2)=\sum r^3-\sum 2$	Attempt two sums	M1
	$\sum r^3-\sum 2n$ **is M0**		
	$=\frac{1}{4}n^2(n+1)^2-2n$	Correct expression	A1
	$=\frac{n}{4}(n^3+2n^2+n-8)$ *	Completion to printed answer with no errors seen.	A1
			(3)
(c)	$\sum_{r=20}^{r=50}(r^3-2)=\frac{50}{4}\times130042-\frac{19}{4}\times7592$	Attempt $S_{50}-S_{20}$ or $S_{50}-S_{19}$ and substitutes into a correct expression at least once.	M1
	$(=1625525-36062)$	Correct numerical expression (unsimplified)	A1
	= 1 589 463	cao	A1
			(3)
(c) Way 2	$\sum_{r=20}^{r=50}(r^3-2)=\sum_{r=20}^{r=50}r^3-\sum_{r=20}^{r=50}(2)=\frac{50^2}{4}\times51^2-\frac{19^2}{4}\times20^2-2\times31$	M1 for ($S_{50}-S_{20}$ or $S_{50}-S_{19}$ for cubes) – (2x30 or 2x31)	**Total 11**
		A1 correct numerical expression	
	=1 589 463	A1	

Question Number	Scheme	Notes	Marks
10.	$\mathrm{f}(n) = 2^{2n-1} + 3^{2n-1}$ is divisible by 5.		
	$\mathrm{f}(1) = 2^1 + 3^1 = 5,$	Shows that $\mathrm{f}(1) = 5$.	B1
	Assume that for $n = k$, $\mathrm{f}(k) = 2^{2k-1} + 3^{2k-1}$ is divisible by 5 for $k \in ¢^{+}$.		
	$\mathrm{f}(k+1) - \mathrm{f}(k) = 2^{2(k+1)-1} + 3^{2(k+1)-1} - \left(2^{2k-1} + 3^{2k-1}\right)$	M1: Attempts $\mathrm{f}(k+1) - \mathrm{f}(k)$. A1: Correct expression for <u>f(k + 1)</u> (Can be unsimplified)	M1A1
	$= 2^{2k+1} + 3^{2k+1} - 2^{2k-1} - 3^{2k-1}$		
	$= 2^{2k-1+2} + 3^{2k-1+2} - 2^{2k-1} - 3^{2k-1}$		
	$= 4\left(2^{2k-1}\right) + 9\left(3^{2k-1}\right) - 2^{2k-1} - 3^{2k-1}$	Achieves an expression in 2^{2k-1} and 3^{2k-1}	M1
	$= 3\left(2^{2k-1}\right) + 8\left(3^{2k-1}\right)$		
	$= 3\left(2^{2k-1}\right) + 3\left(3^{2k-1}\right) + 5\left(3^{2k-1}\right)$		
	$= 3\mathrm{f}(k) + 5\left(3^{2k-1}\right)$		
	$\therefore \mathrm{f}(k+1) = 4\mathrm{f}(k) + 5\left(3^{2k-1}\right)$ or $4(2^{2k-1} + 3^{2k-1}) + 5\left(3^{2k-1}\right)$	Where f(k + 1) is correct and is clearly a multiple of 5.	A1
	If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all *n*.	Correct conclusion **at the end,** at least as given, and all previous marks scored.	A1 cso
			[6]
			6 marks
	All methods should complete to f(k + 1) = ... where f(k +1) is clearly shown to be divisible by 5 to enable the final 2 marks to be available.		
Note that there are many different ways of proving this result by induction.			

Question Number	Scheme	Notes	Marks
***Aliter* 10. Way 2**	$\text{f}(n) = 2^{2n-1} + 3^{2n-1}$ is divisible by 5.		
	$\text{f}(1) = 2^1 + 3^1 = 5$	Shows that $\text{f}(1) = 5$.	B1
	Assume that for $n = k$, $\text{f}(k) = 2^{2k-1} + 3^{2k-1}$ is divisible by 5 for $k \in ¢^+$.		
	$\text{f}(k+1) = 2^{2(k+1)-1} + 3^{2(k+1)-1}$	M1: Attempts $\text{f}(k+1)$. A1: Correct expression for <u>f(k + 1)</u> (Can be unsimplified)	M1A1
	$= 2^{2k+1} + 3^{2k+1}$		
	$= 4\left(2^{2k-1}\right) + 9\left(3^{2k-1}\right)$	Achieves an expression in 2^{2k-1} and 3^{2k-1}	M1
	$\text{f}(k+1) = 4\left(2^{2k-1} + 3^{2k-1}\right) + 5\left(3^{2k-1}\right)$ or $\text{f}(k+1) = 4\text{f}(k) + 5\left(3^{2k-1}\right)$ or $\text{f}(k+1) = 9\text{f}(k) - 5\left(2^{2k-1}\right)$ or $\text{f}(k+1) = 9\left(2^{2k-1} + 3^{2k-1}\right) - 5\left(2^{2k-1}\right)$	Where f(k + 1) is correct and is clearly a multiple of 5.	A1
	If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all *n*.	Correct conclusion **at the end**, at least as given, and all previous marks scored.	A1 cso
			[6]

Question Number	Scheme	Notes	Marks
Aliter **10. Way 3**	$f(n) = 2^{2n-1} + 3^{2n-1}$ is divisible by 5.		
	$f(1) = 2^1 + 3^1 = 5$,	Shows that $f(1) = 5$.	B1
	Assume that for $n = k$, $f(k) = 2^{2k-1} + 3^{2k-1}$ is divisible by 5 for $k \in$ ¢ $^{+}$.		
	$f(k+1) + f(k) = 2^{2(k+1)-1} + 3^{2(k+1)-1} + 2^{2k-1} + 3^{2k-1}$	M1: Attempts $f(k+1) + f(k)$. A1: Correct expression for <u>f(k + 1)</u> (Can be unsimplified)	M1A1
	$= 2^{2k+1} + 3^{2k+1} + 2^{2k-1} + 3^{2k-1}$		
	$= 2^{2k-1+2} + 3^{2k-1+2} + 2^{2k-1} + 3^{2k-1}$		
	$= 4\left(2^{2k-1}\right) + 2^{2k-1} + 9\left(3^{2k-1}\right) + 3^{2k-1}$	Achieves an expression in 2^{2k-1} and 3^{2k-1}	M1
	$= 5\left(2^{2k-1}\right) + 10\left(3^{2k-1}\right)$		
	$= 5\left(2^{2k-1}\right) + 5\left(3^{2k-1}\right) + 5\left(3^{2k-1}\right)$		
	$= 5f(k) + 5\left(3^{2k-1}\right)$		
	$\therefore f(k+1) = 4f(k) + 5\left(3^{2k-1}\right)$ or $4\left(2^{2k-1} + 3^{2k-1}\right) + 5\left(3^{2k-1}\right)$	Where f(k + 1) is correct and is clearly a multiple of 5.	A1
	If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all *n*.	Correct conclusion **at the end**, at least as given, and all previous marks scored.	A1 cso
			[6]
			6 marks

Question Number	Scheme	Notes	Marks
Aliter **10. Way 4**	$\mathrm{f}(n) = 2^{2n-1} + 3^{2n-1}$ is divisible by 5.		
	$\mathrm{f}(1) = 2^1 + 3^1 = 5,$	Shows that $\mathrm{f}(1) = 5$.	B1
	Assume that for $n = k$, $\mathrm{f}(k) = 2^{2k-1} + 3^{2k-1}$ is divisible by 5 for $k \in ¢^+$.		
	$\mathrm{f}(k+1) = \mathrm{f}(k+1) + \mathrm{f}(k) - \mathrm{f}(k)$		
	$\mathrm{f}(k+1) = 2^{2(k+1)-1} + 3^{2(k+1)-1} + 2^{2k-1} + 3^{2k-1} - (2^{2k-1} + 3^{2k-1})$	M1: Attempts $\mathrm{f}(k+1) + \mathrm{f}(k) - \mathrm{f}(k)$ A1: Correct expression for <u>f(k + 1)</u> (Can be unsimplified)	M1A1
	$= 4\left(2^{2k-1}\right) + 9\left(3^{2k-1}\right) + 2^{2k-1} + 3^{2k-1} - \left(2^{2k-1} + 3^{2k-1}\right)$	Achieves an expression in 2^{2k-1} and 3^{2k-1}	M1
	$= 5\left(2^{2k-1}\right) + 10\left(3^{2k-1}\right) - \left(2^{2k-1} + 3^{2k-1}\right)$		
	$= 5\left(\left(2^{2k-1}\right) + 2\left(3^{2k-1}\right)\right) - \left(2^{2k-1} + 3^{2k-1}\right)$		
	$= 5\left(\left(2^{2k-1}\right) + 2\left(3^{2k-1}\right)\right) - \mathrm{f}(k)$ *or* $5\left(\left(2^{2k-1}\right) + 2\left(3^{2k-1}\right)\right) - (2^{2k-1} + 3^{2k-1})$	Where f(k + 1) is correct and is clearly a multiple of 5.	A1
	If the result is true for $n = k$, **then it is now true for** $n = k+1$. **As the result has shown to be true for** $n = 1$, **then the result is true for all *n*.**	Correct conclusion **at the end**, at least as given, and all previous marks scored.	A1 cso
			[6]
			6 marks