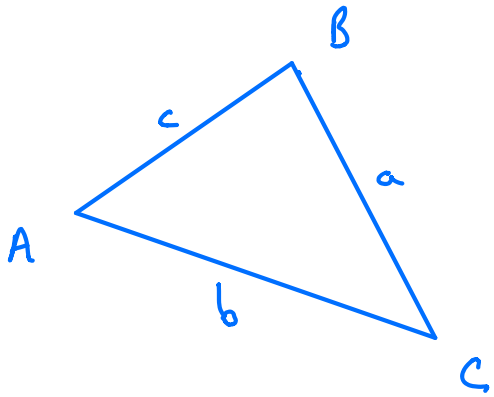
Trigonometry

Sine and Cosine Rules



Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Circular Measure

Degrees	Radians
180°	π
90°	$\frac{\pi}{2}$
60°	$\frac{\pi}{3}$
45°	$\frac{\pi}{4}$
30°	$\frac{\pi}{6}$
0°	0

120°	$\frac{2\pi}{3}$
135°	$\frac{3\pi}{4}$
150°	$\frac{5\pi}{6}$
360°	2π

Radian measure is based on a full circle 360° being equal to 2π radians

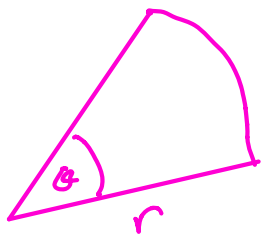
so $1 \text{ radian} = \frac{360}{2\pi} = 57.3^\circ$

Conversion

Degrees $\longrightarrow$ Radians $\times \frac{\pi}{180}$

Radians $\longrightarrow$ Degrees $\times \frac{180}{\pi}$

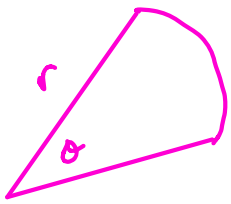
Arc Length of a circle



Arc Length $= r\theta$

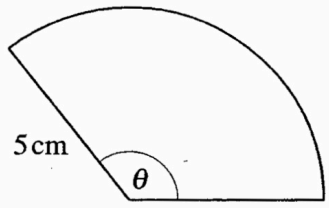
if θ in radians

Area of a sector



Area $= \frac{1}{2} r^2 \theta$

7



Not to scale

Fig. 7

Fig. 7 shows a sector of a circle of radius 5 cm which has angle θ radians. The sector has area 30 cm^2.

(i) Find θ. [3]

(ii) Hence find the perimeter of the sector. [2]

i)

$$\text{Area} = \frac{1}{2}r^2\theta$$

$$30 = \frac{1}{2} \times 5^2 \times \theta$$

$$\frac{60}{25} = \theta$$

$$\theta = 2.4 \text{ radians}$$

ii)

$$\text{Perimeter} = r + r + r\theta$$

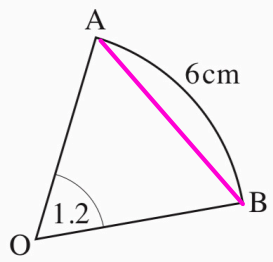
$$= 5 + 5 + 5 \times 2.4$$

$$= 22 \text{ cm}$$

7 In Fig. 7, A and B are points on the circumference of a circle with centre O.

Angle AOB = 1.2 radians.

The arc length AB is 6 cm.



Not to scale

Fig. 7

(i) Calculate the radius of the circle. [2]

(ii) Calculate the length of the chord AB. [3]

i) Arc Length $= r\theta$

$6 = 1.2r$

$r = \frac{6}{1.2}$

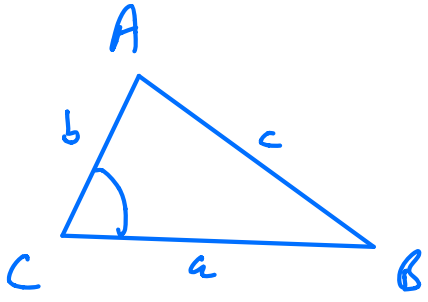
$r = 5$ cm

ii) Cosine rule

$AB^2 = 5^2 + 5^2 - 2 \times 5 \times 5 \cos 1.2$

$AB = 5.65$ cm

Area of a Triangle



Area of a $\triangle$

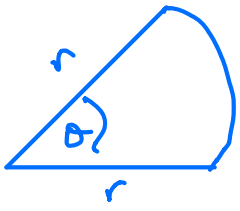
$= \frac{1}{2} ab \sin C$

5 A sector of a circle of radius 5 cm has area 9 cm^2.

Find, in radians, the angle of the sector.

Find also the perimeter of the sector. [5]

$$\frac{1}{2}r^2\theta = \text{Area}$$



$$\frac{1}{2} \times 5^2\theta = 9$$

$$\theta = \frac{18}{25}$$

$$\theta = 0.72 \text{ radians}$$

Perimeter $r + r + r\theta$

$= 5 + 5 + 5 \times 0.72$

$= 13.6$ cm