$$\underline{a} = 3\underline{i} + 4\underline{j} + 5\underline{k}$$

$$|\underline{a}| = \sqrt{3^2 + 4^2 + 5^2} = \sqrt{50}$$

Position vector of $A(3,4,5)$

$$= 3\underline{i} + 4\underline{j} + 5\underline{k} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$$

measured from an origin $O(0,0,0)$

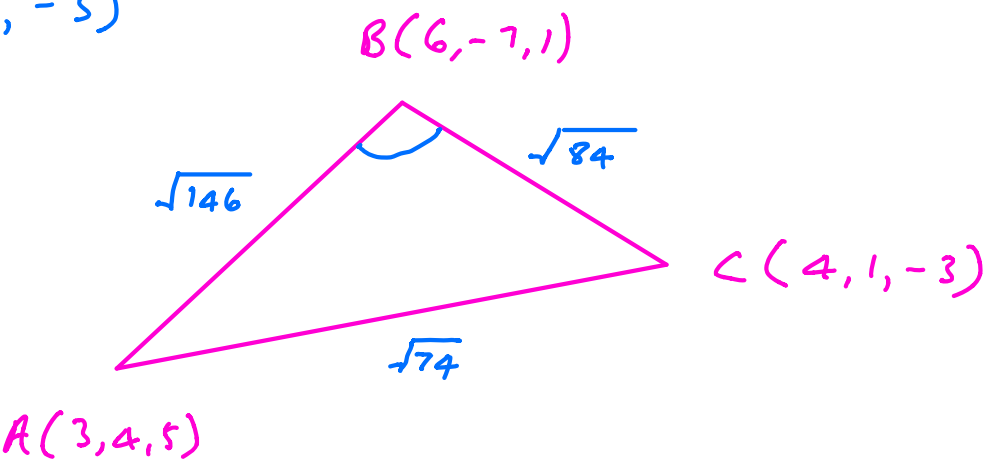
Finding angles between vectors

$A(3,4,5)$

$B(6,-7,1)$

$C(4,1,-3)$

Find $\angle ABC$



$$|AB| = \sqrt{(6-3)^2 + (-7-4)^2 + (1-5)^2} = \sqrt{9 + 121 + 16} = \sqrt{146}$$

$$|BC| = \sqrt{(6-4)^2 + (-7-1)^2 + (1--3)^2} = \sqrt{4 + 64 + 16} = \sqrt{84}$$

$$|AC| = \sqrt{(3-4)^2 + (4-1)^2 + (5--3)^2} = \sqrt{1 + 9 + 64} = \sqrt{74}$$

$$\cos \angle ABC = \frac{84 + 146 - 74}{2\sqrt{146}\sqrt{84}} = \frac{156}{2\sqrt{12264}}$$

$$\cos \angle ABC \approx 0.7043$$

$$\angle ABC = 45.2^\circ$$

Off Syllabus

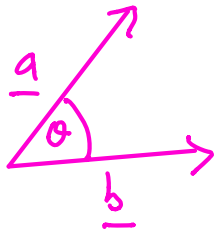
Scalar Product

If $\underline{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\underline{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

The scalar product or dot product of $\underline{a}$ and $\underline{b}$ written as $\underline{a} . \underline{b}$ is given by

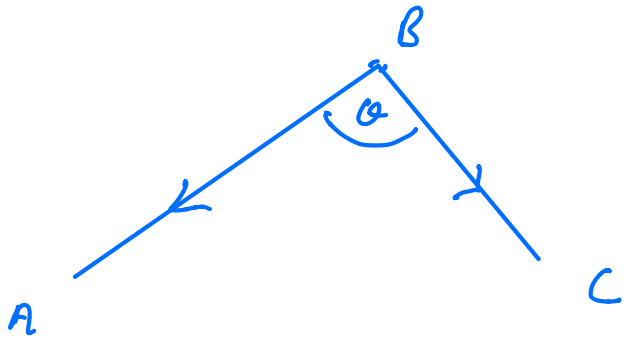
$$\underline{a} . \underline{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

The angle between two vectors $\underline{a}$ and $\underline{b}$ can be found by



$$\cos\theta = \frac{\underline{a} . \underline{b}}{|\underline{a}||\underline{b}|}$$

Going back to our first example



$$\overrightarrow{BA} = \begin{pmatrix} -3 \\ 11 \\ 4 \end{pmatrix} \qquad \overrightarrow{BC} = \begin{pmatrix} -2 \\ 8 \\ -4 \end{pmatrix}$$

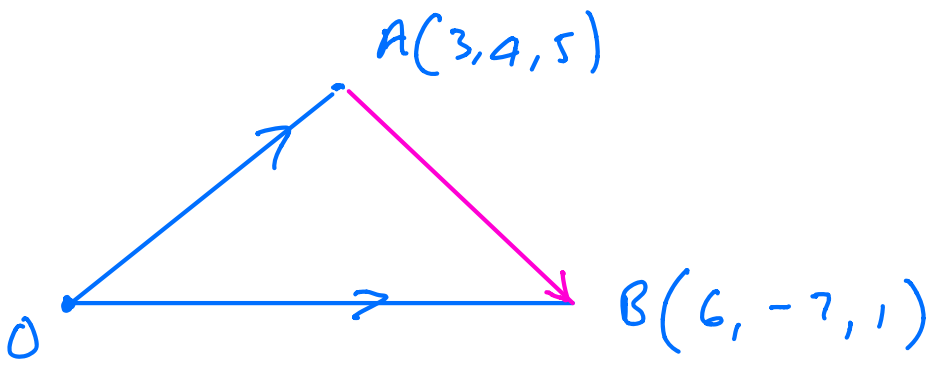
$$\cos \angle ABC = \frac{\begin{pmatrix} -3 \\ 11 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 8 \\ -4 \end{pmatrix}}{\left| \begin{pmatrix} -3 \\ 11 \\ 4 \end{pmatrix} \right| \left| \begin{pmatrix} -2 \\ 8 \\ -4 \end{pmatrix} \right|}$$

$$\cos \angle ABC = \frac{+6 + 88 - 16}{\sqrt{9+121+16}\sqrt{4+64+16}}$$

$$= \frac{78}{\sqrt{146}\sqrt{84}}$$

$$\angle ABC = \cos^{-1}\left(\frac{78}{\sqrt{146}\sqrt{84}}\right) = 45.2^\circ$$

Finding the Vector $\overrightarrow{AB}$



$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$

$$= \begin{pmatrix} -3 \\ -4 \\ -5 \end{pmatrix} + \begin{pmatrix} 6 \\ -7 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -11 \\ -4 \end{pmatrix}$$
