Poisson - Expectation and Variance

Binomial Distribution	Poisson Distribution
$X \sim B(n, p)$	$X \sim Po(\lambda)$
$E(x) = np$	$E(x) = \lambda$
$Var(x) = npq$ or $np(1-p)$	$Var(x) = \lambda$

Exercise 2E

Q1 $\quad \Sigma x = 143 \qquad \Sigma x^2 = 347$

a) $E(x) = \dfrac{143}{100} = 1.43$

$E(x^2) = \dfrac{347}{100} = 3.47$

$Var(x) = E(x^2) - (E(x))^2$

$= 3.47 - 1.43^2 = 1.4251$

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b) Poisson could be suitable since $E(x) \approx Var(x)$

c) $X \sim Po(1.43)$

$$P(x=3) = \frac{e^{-1.43} \times 1.43^3}{3!} = 0.1166$$

Exercise 2F

5) $X \sim B(n, p)$ $\quad E(x) = 4.8$

$Var(x) = 2.88$

$np = 4.8$

$np(1-p) = 2.88$

$4.8(1-p) = 2.88$

$1-p = \frac{2.88}{4.8} = 0.6$

$\underline{p = 0.4}$

$0.4n = 4.8$

$n = \frac{4.8}{0.4}$ $\quad \underline{n = 12}$

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7) $X \sim B(10, 0.65)$

a)

$P(X=5) = {}^{10}C_5 \times 0.65^5 \times 0.35^5$

i) $= 0.1536$

$P(X \geq 5) = 1 - P(X \leq 4)$

$= 1 - 0.0949$

ii) $= 0.9051$

b) i) $np = 78$

$0.65n = 78$

$n = \frac{78}{0.65}$

$\underline{n = 120}$

ii) $Var = npq = 120 \times 0.65 \times 0.35$

$= 27.3$

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a) $X \sim B(5, p)$ $\quad P(X \geq 1) = 0.83193$

$\Rightarrow P(X=0) = 0.16807$

$(1-p)^5 = 0.16807$

$1-p = \sqrt[5]{0.16807} = 0.7$

$\underline{\therefore p = 0.3}$

$E(X) = np = 5 \times 0.3 = 1.5$

$Var(X) = npq = 5 \times 0.3 \times 0.7 = 1.05$

Exercise 2G

5) $X \sim B(\overset{n}{100}, \overset{p}{0.03})$

$E(X) = 100 \times 0.03$
$= 3$

$Var(X) = 3 \times 0.97$
$= 2.91$

Approx with $Y \sim Po(3)$

$P(Y<4) = P(Y \leq 3) = 0.6472$

$P(Y=2) = 0.2240$

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11) a) $X \sim B(\overset{n}{1200}, \overset{p}{0.005})$

b) $E(X) = 1200 \times 0.005 = 6$

$Var(X) = 6 \times 0.995 = 5.97$

c) Approx with $Y \sim Po(6)$

$P(Y \leq 4) = 0.2851$

Exercise 2G Even numbers