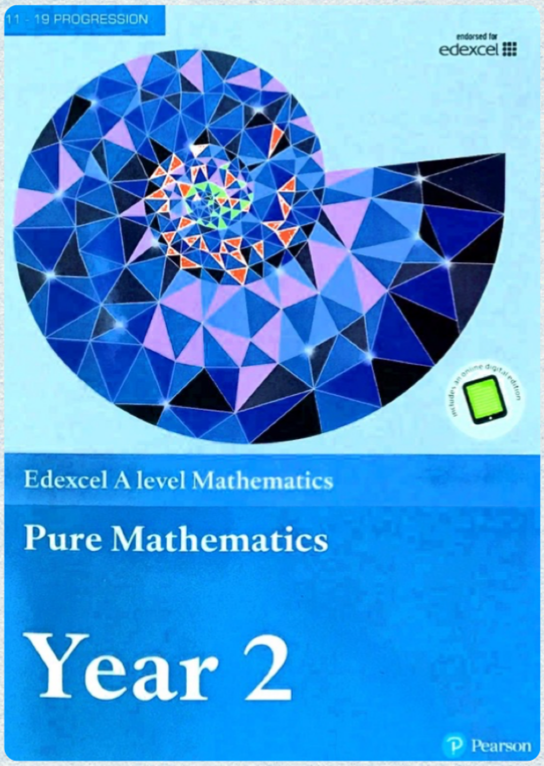

Chapter 1

Algebraic Methods

1.1 Proof by Contradiction

Negation of Statements

(P) **1** Select the statement that is the negation of 'All multiples of three are even'.

A All multiples of three are odd.

B At least one multiple of three is odd.

C No multiples of three are even.

Proof by Contradiction

(P) **3** Statement: If n^2 is odd then n is odd.

a Write down the negation of this statement.

b Prove the original statement by contradiction.

In this question n is an integer

a) There is at least one value of n such that n is even and n^2 is odd

b) Given n^2 is odd assume n is even

n even $\Rightarrow$ $n = 2k$ for some integer k

$n^2 = (2k)^2 = 4k^2$

$n^2 = 2(2k^2)$

n^2 is even Contradiction

$\therefore$ n is odd

Let A be a rational number
Let B be an irrational number
Suppose A + B = C
Prove C is irrational

A can be written as $\frac{p}{q}$ p, q integers

B cannot be written in this way

Assume C is rational

$\therefore \quad C = \frac{h}{k}$ for h, k integers

$$\Rightarrow \quad \frac{p}{q} + B = \frac{h}{k}$$

$$\Rightarrow \quad B = \frac{h}{k} - \frac{p}{q} = \frac{hq - pk}{kq} = \frac{\text{an integer}}{\text{an integer}}$$

$\Rightarrow$ B is rational   Contradiction

$\therefore$ C is irrational since assumption wrong