Vectors in Kinematics and Statics Problems

1 The position vector, $\mathbf{r}$, of a particle of mass 4 kg at time t is given by

$$\mathbf{r} = t^2\mathbf{i} + (5t - 2t^2)\mathbf{j},$$

where $\mathbf{i}$ and $\mathbf{j}$ are the standard unit vectors, lengths are in metres and time is in seconds.

(i) Find an expression for the acceleration of the particle. [4]

The particle is subject to a force $\mathbf{F}$ and a force $12\mathbf{j}$ N.

(ii) Find $\mathbf{F}$. [3]

3 A particle rests on a smooth, horizontal plane. Horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ lie in this plane. The particle is in equilibrium under the action of the three forces $(-3\mathbf{i} + 4\mathbf{j})$ N and $(21\mathbf{i} - 7\mathbf{j})$ N and $\mathbf{R}$ N.

(i) Write down an expression for $\mathbf{R}$ in terms of $\mathbf{i}$ and $\mathbf{j}$. [2]

(ii) Find the magnitude of $\mathbf{R}$ and the angle between $\mathbf{R}$ and the $\mathbf{i}$ direction. [4]

5 The position vector of a particle at time t is given by

$$\mathbf{r} = \tfrac{1}{2}t\mathbf{i} + (t^2 - 1)\mathbf{j},$$

referred to an origin O where $\mathbf{i}$ and $\mathbf{j}$ are the standard unit vectors in the directions of the cartesian axes Ox and Oy respectively.

(i) Write down the value of t for which the x-coordinate of the position of the particle is 2. Find the y-coordinate at this time. [2]

(ii) Show that the cartesian equation of the path of the particle is $y = 4x^2 - 1$. [2]

(iii) Find the coordinates of the point where the particle is moving at 45° to both Ox and Oy. [3]

3 A force $\mathbf{F}$ is given by $\mathbf{F} = (3.5\mathbf{i} + 12\mathbf{j})$ N, where $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors east and north respectively.

(i) Calculate the magnitude of $\mathbf{F}$ and also its direction as a bearing. [3]

(ii) $\mathbf{G}$ is the force $(7\mathbf{i} + 24\mathbf{j})$ N. Show that $\mathbf{G}$ and $\mathbf{F}$ are in the same direction and compare their magnitudes. [2]

(iii) Force $\mathbf{F}_1$ is $(9\mathbf{i} - 18\mathbf{j})$ N and force $\mathbf{F}_2$ is $(12\mathbf{i} + q\mathbf{j})$ N. Find q so that the sum $\mathbf{F}_1 + \mathbf{F}_2$ is in the direction of $\mathbf{F}$. [2]

5 The acceleration of a particle of mass 4 kg is given by $\mathbf{a} = (9\mathbf{i} - 4t\mathbf{j})\ \text{m s}^{-2}$, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors and t is the time in seconds.

(i) Find the acceleration of the particle when $t = 0$ and also when $t = 3$. [1]

(ii) Calculate the force acting on the particle when $t = 3$. [1]

The particle has velocity $(4\mathbf{i} + 2\mathbf{j})\ \text{m s}^{-1}$ when $t = 1$.

(iii) Find an expression for the velocity of the particle at time t. [4]

2 Force $\mathbf{F}_1$ is $\begin{pmatrix} -6 \\ 13 \end{pmatrix}$N and force $\mathbf{F}_2$ is $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$N, where $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are vectors east and north respectively.

(i) Calculate the magnitude of $\mathbf{F}_1$, correct to three significant figures. [2]

(ii) Calculate the direction of the force $\mathbf{F}_1 - \mathbf{F}_2$ as a bearing. [3]

Force $\mathbf{F}_2$ is the resultant of all the forces acting on an object of mass 5 kg.

(iii) Calculate the acceleration of the object and the change in its velocity after 10 seconds. [3]

4 Fig. 4 shows the unit vectors $\mathbf{i}$ and $\mathbf{j}$ in the directions of the cartesian axes Ox and Oy, respectively. O is the origin of the axes and of position vectors.

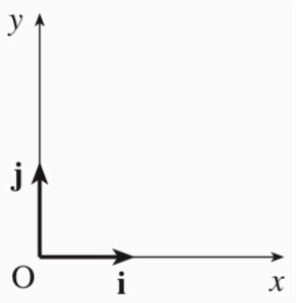

Fig. 4

The position vector of a particle is given by $\mathbf{r} = 3t\mathbf{i} + (18t^2 - 1)\mathbf{j}$ for $t \geqslant 0$, where t is time.

(i) Show that the path of the particle cuts the x-axis just once. [2]

(ii) Find an expression for the velocity of the particle at time t.

Deduce that the particle never travels in the $\mathbf{j}$ direction. [3]

(iii) Find the cartesian equation of the path of the particle, simplifying your answer. [3]