1 A small firework is fired from a point O at ground level over horizontal ground. The highest point reached by the firework is a horizontal distance of 60 m from O and a vertical distance of 40 m from O, as shown in Fig. 7. Air resistance is negligible.

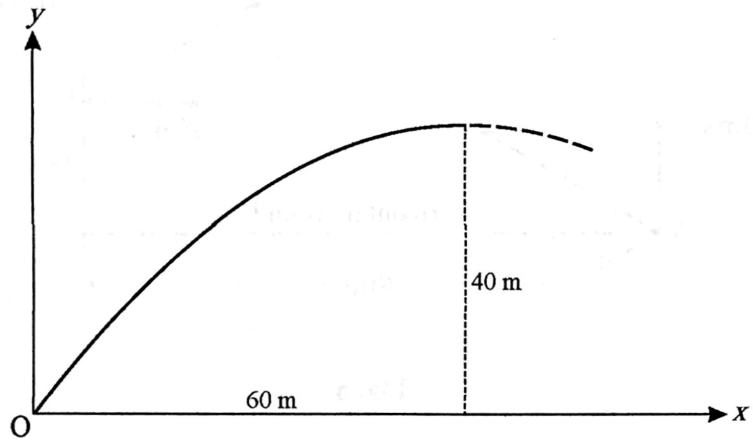

Fig. 7

The initial horizontal component of the velocity of the firework is 21 m s^{-1}.

(i) Calculate the time for the firework to reach its highest point and show that the initial vertical component of its velocity is 28 m s^{-1}. [4]

(ii) Show that the firework is $(28t - 4.9t^2)$ m above the ground t seconds after its projection. [1]

When the firework is at its highest point it explodes into several parts. Two of the parts initially continue to travel horizontally in the original direction, one with the original horizontal speed of 21 m s^{-1} and the other with a quarter of this speed.

(iii) State why the two parts are always at the same height as one another above the ground and hence find an expression in terms of t for the distance between the parts t seconds after the explosion. [3]

(iv) Find the distance between these parts of the firework

(A) when they reach the ground, [2]

(B) when they are 10 m above the ground. [5]

(v) Show that the cartesian equation of the trajectory of the firework before it explodes is $y = \frac{1}{90}(120x - x^2)$, referred to the coordinate axes shown in Fig. 7. [4]

a) Time to travel 60m horizontally

$$= \frac{60}{21} = \frac{20}{7} \text{ s}$$

So time to highest point $= \frac{20}{7}$ s or 2.86 s

$$v_y = u_y + at$$

At top $\quad 0 = u_y - 9.8 \times \frac{20}{7}$

$$u_y = 9.8 \times \frac{20}{7} = 28 \text{ ms}^{-1}$$

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ii) $y = u_y t + \frac{1}{2}at^2$

$$y = 28t - \frac{1}{2} \times 9.8t^2$$

$$y = 28t - 4.9t^2$$

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iii) After explosion both travelling horizontally with no vertical initial velocity. Both subject to same downward acceleration due to gravity so always at the same vertical height as each other.

Horizontal speeds 21 ms^{-1} and $\frac{21}{4} \text{ ms}^{-1}$

Relative speed of fast one to slow one

$$= 21 - \frac{21}{4} = 15.75 \text{ ms}^{-1}$$

Distance apart after t s $= 15.75t$ m

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iv) A) Reach ground in $\frac{20}{7}$ s

Distance apart $15.75 \times \frac{20}{7} = 45\text{ m}$

B) when $y = 10\text{m}$

$s = ut + \frac{1}{2}at^2$

Loss in height $= 0 + 4.9t^2$

$30 = 4.9t^2$

$\frac{30}{4.9} = t^2$

$2.47436 = t$

Distance apart $= 15.75 \times 2.47436 = 38.97$

$= 39.0\text{ m}$

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v) $x = u_x t = 21t \Rightarrow t = \frac{x}{21}$

$y = u_y t + \frac{1}{2}at^2$

$y = 28t - 4.9t^2$

Sub for t

$y = 28\frac{x}{21} - 4.9\left(\frac{x}{21}\right)^2$

$y = \frac{4x}{3} - \frac{x^2}{90}$

$y = \frac{1}{90}\left(120x - x^2\right)$

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2 In this question the value of g should be taken as $10\ \mathrm{m\,s}^{2}$.

As shown in Fig. 8, particles A and B are projected towards one another. Each particle has an initial speed of $10\ \mathrm{m\,s}^{1}$ vertically and $20\ \mathrm{m\,s}^{1}$ horizontally. Initially A and B are 70 m apart horizontally and B is 15 m higher than A. Both particles are projected over horizontal ground.

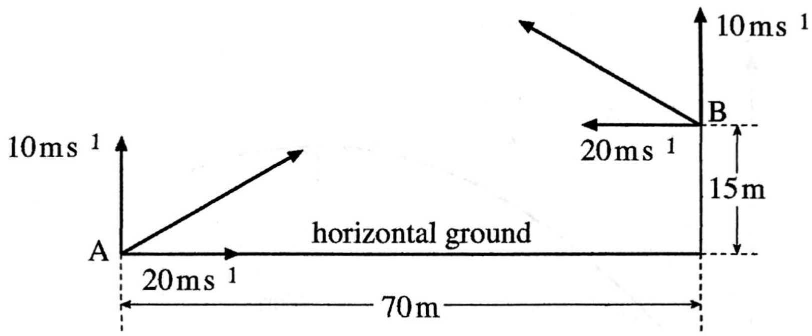

Fig. 8

(i) Show that, t seconds after projection, the height in metres of each particle above its point of projection is $10t - 5t^2$. [1]

(ii) Calculate the horizontal range of A. Deduce that A hits the horizontal ground between the initial positions of A and B. [5]

(iii) Calculate the horizontal distance travelled by B before reaching the ground. [5]

(iv) Show that the paths of the particles cross but that the particles do not collide if they are projected at the same time. [2]

In fact, particle A is projected 2 seconds after particle B.

(v) Verify that the particles collide 0.75 seconds after A is projected. [5]

i) Vert $s = ut + \frac{1}{2}at^2$

$y = 10t - \frac{1}{2} \times 10t^2$

$y = 10t - 5t^2$

where y is height above launch point

ii) $y = 10t - 5t^2$ for A

lands when $y = 0$ $\Rightarrow$ $0 = 10t - 5t^2$

$$0 = 5t(2-t)$$
$$t = 0 \quad \text{or} \quad t = 2$$

Time of flight $= 2s$

Horizontal Range $= U_x t = 20 \times 2 = 40m$

$40m < 70m \therefore$ lands between A and B

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iii) B hits ground when $y = -15$

$$s = ut + \tfrac{1}{2}at^2$$
$$y = 10t - 5t^2$$
$$-15 = 10t - 5t^2$$
$$5t^2 - 10t - 15 = 0$$
$$t^2 - 2t - 3 = 0$$
$$(t+1)(t-3) = 0$$
$$\cancel{t = -1} \quad \text{or} \quad t = 3$$

$$x = U_x t = 20 \times 3 = 60m$$

B travels 60m horizontally before hitting ground

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iv) Since A travels 40m horizontally
and B travels 60m horizontally
they both pass midpoint horizontally 35m from origin

Same horizontal speed so both reach horizontal midpoint at $\frac{35}{20} = 1.75s$

A Height above ground $y = 10t - 5t^2$
$= 10 \times 1.75 - 5 \times 1.75^2$

$= 2.1875$ m

B Height above ground $= 15 + 2.1875$ m

$= 17.1875$ m

$\therefore$ B is 15m above A when they cross

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v) At time $t = 2.75$ for B

$x = 70 - 20 \times 2.75$

relative to O $\quad x = 15$ m

At $t = 2.75$ $\quad y = 10t - 5t^2 + 15$

relative to O $\quad y = 10 \times 2.75 - 5 \times 2.75^2 + 15$

$y = 4.6875$ m

B is at $(15, 4.6875)$

For A time $t = 0.75$ s

$x = 20 \times 0.75 = 15$ m

$y = 10t - 5t^2$

$= 10 \times 0.75 - 5 \times 0.75^2 = 4.6875$ m

A also at $(15, 4.6875)$

Particles $\therefore$ collide
