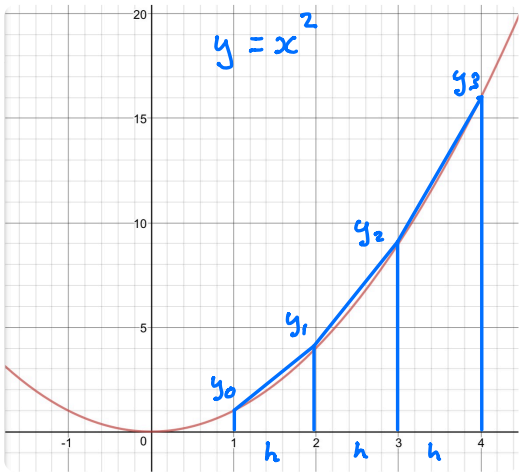
Trapezium Rule



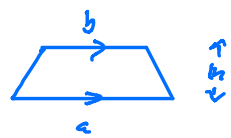
Exact Area

$$A = \int_1^4 x^2\,dx$$

$$= \left[\frac{x^3}{3}\right]_1^4$$

$$= \frac{4^3}{3} - \frac{1^3}{3} = 21$$

Area of Trapezium $= \frac{1}{2}(a+b)h$



$$\text{Area} = \frac{1}{2}(y_0+y_1)h + \frac{1}{2}(y_1+y_2)h + \frac{1}{2}(y_2+y_3)h$$

$$= \frac{h}{2}\left[y_0 + y_1 + y_1 + y_2 + y_2 + y_3\right]$$

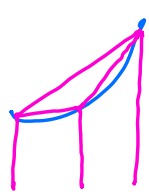
$$= \frac{h}{2}\left[y_0 + 2(y_1+y_2) + y_3\right]$$

n strips $\quad \text{Area} \approx \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + \dots + y_{n-1}) + y_n\right]$

$$\text{Area} = \frac{1}{2}\left[1 + 2(4+9) + 16\right]$$

$$= \frac{43}{2} = 21.5$$

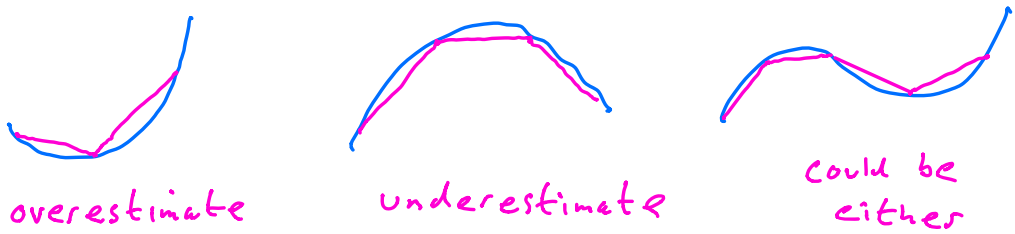
Double number of strips



$$A \approx \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + \dots y_5) + y_6\right]$$

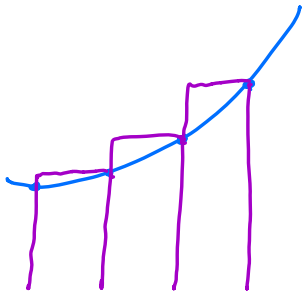
$$= \frac{0.5}{2}\left[1 + 2(2.25 + 4 + 6.25 + 9 + 12.25) + 16\right]$$

$= 21.125$



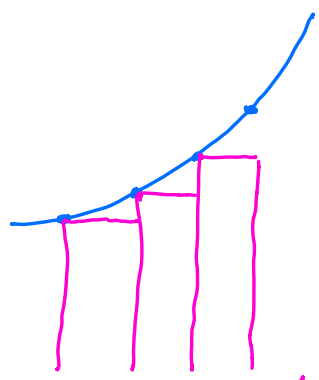
In general, increasing the number of strips will increase the accuracy.

It is also possible to estimate the area using rectangles instead of trapezia. Not as accurate for a given number of strips but you can increase the number of strips to gain more accuracy.



overestimate

taking higher values in each consecutive pair



underestimate

taking lower values in each consecutive pair