Differentiation

Exercise 9F

1 a) $y = \tan 3x \qquad \frac{dy}{dx} = 3\sec^2 3x \quad \text{or} \quad \frac{3}{\cos^2 3x}$

1 b) $y = 4\tan^3 x \qquad \frac{dy}{dx} = 12\tan^2 x \sec^2 x$

$y = 4(\tan x)^3$

1 c) $y = \tan(x-1) \qquad \frac{dy}{dx} = \sec^2(x-1)$

1 d) $y = x^2 \tan\frac{x}{2} + \tan\left(x - \frac{1}{2}\right)$

$$\frac{dy}{dx} = \frac{1}{2}x^2 \sec^2\frac{x}{2} + 2x\tan\frac{x}{2} + \sec^2\left(x - \frac{1}{2}\right)$$

3a $f(x) = (\sec x)^{\frac{1}{2}} = \left(\frac{1}{\cos x}\right)^{\frac{1}{2}}$

$$f'(x) = \frac{1}{2}\left(\frac{1}{\cos x}\right)^{-\frac{1}{2}} \frac{\sin x}{\cos^2 x}$$

[Aside $\frac{d}{dx}\frac{1}{\cos x} = \frac{d}{dx}(\cos x)^{-1} = -1(\cos x)^{-2}(-\sin x)$

$= \frac{\sin x}{\cos^2 x}$]

$$f'(x) = \frac{1}{2}(\cos x)^{\frac{1}{2}} \frac{\sin x}{\cos^2 x} = \frac{\sin x}{2(\cos x)^{3/2}}$$

3b) $f(x) = \sqrt{\cot x} = (\cot x)^{\frac{1}{2}}$

$$f'(x) = -\frac{1}{2}(\cot x)^{-\frac{1}{2}} \operatorname{cosec}^2 x$$

[Aside $\frac{d}{dx} \cot x = \frac{d}{dx} \frac{\cos x}{\sin x} = \frac{\sin x(-\sin x) - \cos x \cos x}{\sin^2 x}$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= -\operatorname{cosec}^2 x$$]

$$f'(x) = -\frac{1}{2}(\tan x)^{\frac{1}{2}} \operatorname{cosec}^2 x$$

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5) $y = \dfrac{1}{\cos x \sin x}$ $\quad 0 < x \leq \pi$

TRIG DOUBLE ANGLES

$$\sin 2x = 2 \sin x \cos x$$

$$\left.\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ \text{or} \quad & 1 - 2\sin^2 x \\ \text{or} \quad & 2\cos^2 x - 1 \end{aligned}\right\}$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$y = \frac{2}{\sin 2x} = 2(\sin 2x)^{-1}$$

a) $$\frac{dy}{dx} = -2(\sin 2x)^{-2} \times 2\cos 2x$$

$$\frac{dy}{dx} = -\frac{4\cos 2x}{\sin^2 2x}$$

b) st pt when $-4\cos 2x = 0$

$$\Rightarrow \cos 2x = 0$$

$$\Rightarrow 2x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}$$

2 stationary points

c) when $x = \frac{\pi}{3}$, $y = \frac{2}{\sin\frac{2\pi}{3}} = \frac{2}{\frac{\sqrt{3}}{2}} = \frac{4}{\sqrt{3}}$

$\frac{dy}{dx}$ when $x = \frac{\pi}{3}$, $\frac{dy}{dx} = \frac{-4\cos\frac{2\pi}{3}}{\sin^2\frac{2\pi}{3}}$

$$\frac{dy}{dx} = \frac{-4(-\frac{1}{2})}{\frac{3}{4}} = \frac{8}{3}$$

tgt $y - y_1 = m(x - x_1)$

$$y - \frac{4}{\sqrt{3}} = \frac{8}{3}\left(x - \frac{\pi}{3}\right)$$

$\times 3\sqrt{3}$ $$3\sqrt{3}y - 12 = 8\sqrt{3}\left(x - \frac{\pi}{3}\right)$$

$$3\sqrt{3}y - 12 = 8\sqrt{3}x - \frac{8\pi\sqrt{3}}{3}$$

$$8\sqrt{3}x - 3\sqrt{3}y + 12 - \frac{8\pi\sqrt{3}}{3} = 0$$