

## Standard Differentials

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

## Chain Rule

Ex1 Let  $y = (2x^2 + 3x - 5)^3$  Find  $\frac{dy}{dx}$

$$\text{Let } u = 2x^2 + 3x - 5$$

$$\Rightarrow \frac{du}{dx} = 4x + 3$$

$$y = u^3$$

$$\Rightarrow \frac{dy}{du} = 3u^2$$

Chain Rule  $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$\frac{dy}{dx} = 3u^2 \times (4x+3) = 3(2x^2 + 3x - 5)^2(4x+3)$$

This is a formal approach to the Chain Rule

Informal approach

$\frac{d}{dx}$  of a bracket to the power  $n$

=  $n()$  <sup>$n-1$</sup>  × differential of the bracket

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Ex 2

$$y = \sin 4x$$

Find  $\frac{dy}{dx}$

Let  $u = 4x$

$$y = \sin u$$

$$\frac{du}{dx} = 4$$

$$\frac{dy}{du} = \cos u$$

Chain rule  $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$= \cos u \times 4$$

$$= 4 \cos 4x$$

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Ex 3

$$y = \cos(x^4)$$

Find  $\frac{dy}{dx}$

Let  $u = x^4$

$$y = \cos u$$

$$\frac{du}{dx} = 4x^3$$

$$\frac{dy}{du} = -\sin u$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = -\sin u \times 4x^3 = -4x^3 \sin(x^4)$$

Ex 3

Let  $y = e^{x^2+x}$  Find  $\frac{dy}{dx}$

Let  $u = x^2+x \quad y = e^u$

$$\frac{du}{dx} = 2x+1 \quad \frac{dy}{du} = e^u$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} = e^u \times (2x+1) \\ &= (2x+1)e^{x^2+x}\end{aligned}$$

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Informally  $\frac{d}{dx} e^{f(x)} = f'(x) e^{f(x)}$

$$\frac{d}{dx} \sin(f(x)) = f'(x) \cos(f(x))$$

$$\frac{d}{dx} \cos(f(x)) = -f'(x) \sin(f(x))$$

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Ex 4

Let  $y = \ln(x^2+3x+1)$  Find  $\frac{dy}{dx}$

Let  $u = x^2+3x+1 \quad y = \ln u$

$$\frac{du}{dx} = 2x+3 \quad \frac{dy}{du} = \frac{1}{u}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{u} \times (2x+3) = \frac{2x+3}{x^2+3x+1}$$

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$$\text{In general } \frac{d}{dx} \ln(f(x)) = \frac{f'(x)}{f(x)}$$

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$$\begin{aligned} 1b) \quad \frac{d}{dx} (3-2x^2)^{-5} &= -5(3-2x^2)^{-6}(-4x) \\ &= \frac{20x}{(3-2x^2)^6} \end{aligned}$$

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$$\begin{aligned} 1d) \quad \frac{d}{dx} (6x+x^2)^7 &= 7(6x+x^2)^6(2x) \\ &= 14x(6x+x^2)^6 \end{aligned}$$

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$$\begin{aligned} 1f) \quad \frac{d}{dx} \sqrt{7-x} &= \frac{d}{dx} (7-x)^{\frac{1}{2}} \\ &= \frac{1}{2}(7-x)^{-\frac{1}{2}}(-1) \\ &= -\frac{1}{2}(7-x)^{-\frac{1}{2}} \\ \text{or} \quad &- \frac{1}{2\sqrt{7-x}} \end{aligned}$$

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$$\begin{aligned} 1h) \quad \frac{d}{dx} 3(8-x)^{-6} &= 3x - 6(8-x)^{-7}(-1) \\ &= 18(8-x)^{-7} \end{aligned}$$

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$$2b) \frac{d}{dx} \cos(2x-1) = -2 \sin(2x-1)$$

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$$2d) \frac{d}{dx} (\sin x + \cos x)^5 = 5(\sin x + \cos x)^4 (\cos x - \sin x)$$

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$$2f) \frac{d}{dx} \ln \sin x = \frac{\cos x}{\sin x}$$

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$$2h) \frac{d}{dx} \cos(e^{2x} + 3) = -2e^{2x} \sin(e^{2x} + 3)$$

$$4) y = (5-2x)^3$$

$$\frac{dy}{dx} = 3(5-2x)^2(-2)$$

$$\frac{dy}{dx} = -6(5-2x)^2$$

$$\text{when } x=1, \frac{dy}{dx} = -6(5-2)^2 = -54$$

$$\text{when } x=1, y = (5-2)^3 = 27$$

$$m = -54 \quad (x_1, y_1) = (1, 27)$$

$$y - y_1 = m(x - x_1)$$

$$y - 27 = -54(x - 1)$$

$$y = -54x + 54 + 27$$

$$\underline{y = -54x + 81}$$