

Question Number	Scheme	Marks
Q5 (a)	$-(25 - x^2)^{\frac{1}{2}} (+c)$	M1A1 (2)

Question Number	Scheme	Marks
2.	$x^2 + 4x + 13 = (x+2)^2 + 9$ $\int \frac{1}{(x+2)^2 + 9} dx = \frac{1}{3} \arctan\left(\frac{x+2}{3}\right)$ $\left[\frac{1}{3} \arctan\left(\frac{x+2}{3}\right) \right]_{-2}^1 = \frac{1}{3} (\arctan 1 - \arctan 0)$ $= \frac{\pi}{12}$	B1 M1 A1 M1 A1 (5)
		5

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Further Pure Mathematics FP3 6669
Mark Scheme

2.		
(a) (i)	$\frac{dy}{dx} = \frac{x}{\sqrt{(1-x^2)}} + \arcsin x$	M1 A1 (2)
(ii)	At given value derivative $= \frac{1}{\sqrt{3}} + \frac{\pi}{6} = \frac{2\sqrt{3} + \pi}{6}$	B1 (1)
(b)	$\begin{aligned}\frac{dy}{dx} &= \frac{6e^{2x}}{1+9e^{4x}} \\ &= \frac{6}{e^{-2x}+9e^{2x}} \\ &= \frac{3}{\frac{5}{2}(e^{2x}+e^{-2x}) + \frac{4}{2}(e^{2x}-e^{-2x})} \\ \therefore \frac{dy}{dx} &= \frac{3}{5\cosh 2x + 4\sinh 2x}\end{aligned}$ *	1M1 A1 2M1 3M1 A1 cso (5) 8
(a) M1	<u>Notes:</u> Differentiating getting an \arcsinx term and a $\frac{1}{\sqrt{1 \pm x^2}}$ term	
A1	CAO	
B1	CAO any correct form	

Question Number	Scheme	Marks
(b) 1M1 1A1 2M1 3M1 2A1	Of correct form $\frac{ae^{2x}}{1 \pm be^{4x}}$ CAO Getting from expression in e^{4x} to e^{2x} and e^{-2x} only Using sinh2x and cosh2x in terms of $(e^{2x} + e^{-2x})$ and $(e^{2x} - e^{-2x})$ CSO – answer given	
3.	(a) $x^2 - 10x + 34 = (x-5)^2 + 9$ so $\frac{1}{x^2 - 10x + 34} = \frac{1}{(x-5)^2 + 9} = \frac{1}{u^2 + 9}$ (mark can be earned in either part (a) or (b)) $I = \int \frac{1}{u^2 + 9} du = \left[\frac{1}{3} \arctan\left(\frac{u}{3}\right) \right]$ $I = \int \frac{1}{(x-5)^2 + 9} du = \left[\frac{1}{3} \arctan\left(\frac{x-5}{3}\right) \right]$ Uses limits 3 and 0 to give $\frac{\pi}{12}$ Uses limits 8 and 5 to give $\frac{\pi}{12}$	B1 M1 A1 DM1 A1 (5)
(b) Alt 1 (b) Alt 2 (b) Alt 3	$I = \ln\left(\left(\frac{x-5}{3}\right) + \sqrt{\left(\frac{x-5}{3}\right)^2 + 1}\right)$ or $I = \ln\left(\frac{x-5 + \sqrt{(x-5)^2 + 9}}{3}\right)$ or $I = \ln\left((x-5) + \sqrt{(x-5)^2 + 9}\right)$ Uses limits 5 and 8 to give $\ln(1 + \sqrt{2})$. $I = \int \frac{1}{\sqrt{u^2 + 9}} du = \left[\operatorname{arsinh}\left(\frac{u}{3}\right) \right] = \ln\left(u + \sqrt{u^2 + 9}\right)$ Uses limits 3 and 0 and ln expression to give $\ln(1 + \sqrt{2})$. Use substitution $x-5 = 3\tan\theta$, $\frac{dx}{d\theta} = 3\sec^2\theta$ and so $I = \int \sec\theta d\theta = \ln(\sec\theta + \tan\theta)$ Uses limits 0 and $\frac{\pi}{4}$ to get $\ln(1 + \sqrt{2})$.	M1 A1 DM1 A1 (4) M1 A1 DM1 A1 (4) M1 A1 DM1 A1 (4)
(a) B1 1M1 1A1 2DM1 2A1	<u>Notes:</u> CAO allow recovery in (b) Integrating getting k arctan term CAO Correctly using limits. CAO	

Question Number	Scheme	Marks
(b) 1M1 1A1 2DM1 2A1	Integrating to get a ln or hyperbolic term CAO Correctly using limits. CAO	

Question Number	Scheme	Marks
2. (a)	$k \operatorname{arsinh}\left(\frac{2x}{3}\right) (+c)$ or $k \ln[px + \sqrt{(p^2 x^2 + \frac{9}{4} p^2)}] (+c)$	M1
	$\frac{1}{2} \operatorname{ar sinh}\left(\frac{2x}{3}\right) (+c)$ or $\frac{1}{2} \ln[px + \sqrt{(p^2 x^2 + \frac{9}{4} p^2)}] (+c)$	A1
		(2)
(b)	So: $\frac{1}{2} \ln[6 + \sqrt{45}] - \frac{1}{2} \ln[-6 + \sqrt{45}] = \frac{1}{2} \ln\left[\frac{6 + \sqrt{45}}{-6 + \sqrt{45}}\right]$ Uses correct limits and combines logs	M1
	$= \frac{1}{2} \ln\left[\frac{6 + \sqrt{45}}{-6 + \sqrt{45}}\right]\left[\frac{6 + \sqrt{45}}{6 + \sqrt{45}}\right] = \frac{1}{2} \ln\left[\frac{(6 + \sqrt{45})^2}{9}\right]$	M1
	Correct method to rationalise denominator (may be implied) Method must be clear if answer does not follow their fraction	
	$= \ln[2 + \sqrt{5}] \text{ (or } \frac{1}{2} \ln[9 + 4\sqrt{5}] \text{)}$	A1cs0
	Note that the last 3 marks can be scored without the need to rationalise e.g. $2 \times \frac{1}{2} \left[\ln[2x + \sqrt{(4x^2 + 9)}] \right]_0^3 = \ln(6 + \sqrt{45}) - \ln 3 = \ln\left(\frac{6 + \sqrt{45}}{3}\right)$ M1: Uses the limits 0 and 3 and doubles M1: Combines Logs A1: $\ln[2 + \sqrt{5}] \text{ oe}$	(3)
		Total 5
	Alternative for (a) $x = \frac{3}{2} \sinh u \Rightarrow \int \frac{1}{\sqrt{9 \sinh^2 u + 9}} \cdot \frac{3}{2} \cosh u du = k \operatorname{arsinh}\left(\frac{2x}{3}\right) (+c)$	M1
(b)	$\frac{1}{2} \operatorname{ar sinh}\left(\frac{2x}{3}\right) (+c)$	A1
	$\left[\frac{1}{2} \operatorname{arsinh}\left(\frac{2x}{3}\right) \right]_{-3}^3 = \frac{1}{2} \operatorname{arsinh} 2 - \frac{1}{2} \operatorname{arsinh} -2$	
	$\frac{1}{2} \ln(2 + \sqrt{5}) - \frac{1}{2} \ln(\sqrt{5} - 2) = \frac{1}{2} \ln\left(\frac{2 + \sqrt{5}}{\sqrt{5} - 2}\right)$	M1
	Uses correct limits and combines logs	
	$= \frac{1}{2} \ln\left(\frac{2 + \sqrt{5}}{\sqrt{5} - 2} \cdot \frac{\sqrt{5} + 2}{\sqrt{5} + 2}\right) = \frac{1}{2} \ln\left(\frac{2\sqrt{5} + 4 + 5 + 2\sqrt{5}}{5 - 4}\right)$	M1
	Correct method to rationalise denominator (may be implied) Method must be clear if answer does not follow their fraction	
	$= \frac{1}{2} \ln[9 + 4\sqrt{5}]$	A1cs0