

## Trigonometry Addition Formulae

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

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## Double Angle Formulae

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \begin{cases} \cos^2 A - \sin^2 A \\ 1 - 2 \sin^2 A \\ 2 \cos^2 A - 1 \end{cases}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

$$\tan \theta \quad 0 \quad \frac{1}{\sqrt{3}} \quad 1 \quad \frac{\sqrt{3}}{1} \quad \infty$$

$$\begin{aligned} \text{Evaluate } \cos 75^\circ &= \cos(45 + 30) \\ &= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ \\ &= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \\ &= \frac{\sqrt{3} - 1}{2\sqrt{2}} \end{aligned}$$

$$\begin{aligned} \text{Evaluate } \sin 15^\circ &= \sin(45 - 30) \\ &= \sin 45 \cos 30 - \cos 45 \sin 30 \\ &= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \\ &= \frac{\sqrt{3} - 1}{2\sqrt{2}} \end{aligned}$$

$$\begin{aligned} \text{Evaluate } \tan 105^\circ &= \tan(60 + 45) \\ &= \frac{\tan 60 + \tan 45}{1 - \tan 60 \tan 45} \\ &= \frac{\sqrt{3} + 1}{1 - \sqrt{3} \times 1} \\ &= \frac{1 + \sqrt{3}}{1 - \sqrt{3}} \end{aligned}$$

## Homework for Monday

1 Use the compound-angle formulae to write the following as surds.

(i)  $\sin 75^\circ = \sin(45^\circ + 30^\circ)$

(ii)  $\cos 135^\circ = \cos(90^\circ + 45^\circ)$

(iii)  $\tan 15^\circ = \tan(45^\circ - 30^\circ)$

(iv)  $\tan 75^\circ = \tan(45^\circ + 30^\circ)$

2 Expand each of the following expressions.

(i)  $\sin(\theta + 45^\circ)$

(ii)  $\cos(\theta - 30^\circ)$

(iii)  $\sin(60^\circ - \theta)$

(iv)  $\cos(2\theta + 45^\circ)$

(v)  $\tan(\theta + 45^\circ)$

(vi)  $\tan(\theta - 45^\circ)$

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