Intro to Matrices

Examples

$$\underline{A} = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}_{2\times 2}$$

$$\underline{B} = \begin{pmatrix} 1 & 0 & 3 \\ 4 & 1 & 5 \end{pmatrix}_{2\times 3}$$

$$\underline{C} = \begin{pmatrix} 1 & 4 \\ 2 & 0 \\ 0 & 1 \end{pmatrix}_{3\times 2}$$

$$\underline{D} = \begin{pmatrix} 1 & 7 \\ 3 & 2 \end{pmatrix}_{2\times 2}$$

Addition

$$\underline{A} + \underline{D} = \begin{pmatrix} 2+1 & 3+7 \\ 1+3 & 4+2 \end{pmatrix}_{2\times 2}$$

$$= \begin{pmatrix} 3 & 10 \\ 4 & 6 \end{pmatrix}_{2\times 2}$$

Subtraction

$$\underline{A} - \underline{D} = \begin{pmatrix} 2-1 & 3-7 \\ 1-3 & 4-2 \end{pmatrix}_{2\times 2} = \begin{pmatrix} 1 & -4 \\ -2 & 2 \end{pmatrix}_{2\times 2}$$

$\underline{A} + \underline{B}$ is not defined as matrices are not the same size

We say $\underline{A}$ and $\underline{B}$ are non-conformable under addition or subtraction

Multiplication is defined only when the first matrix has the same number of columns as the second one has rows

If $\underline{A}_{m\times p}$ and $\underline{B}_{p\times n}$ then $\underline{A}\,\underline{B}_{m\times n}$

$$\underline{B}\,\underline{C} = \begin{pmatrix}1&0&3\\4&1&5\end{pmatrix}\begin{pmatrix}1&4\\2&0\\0&1\end{pmatrix} = \begin{array}{cc} & \begin{array}{cc} C_1 & \quad C_2\end{array}\\ \begin{array}{c}R_1\\ R_2\end{array} & \begin{pmatrix}\overset{1\times1+0\times2+3\times0}{1} & 7\\ 6 & 21\end{pmatrix}_{2\times2}\end{array}$$

By calc $\underline{C}\,\underline{B} = \begin{pmatrix}17&4&23\\2&0&6\\4&1&5\end{pmatrix}$

NOT EVEN THE SAME SIZE AS $\underline{B}\,\underline{C}$!!

In examples above

$\underline{C}\,\underline{A}$ exists but $\underline{A}\,\underline{C}$ does not

because $\underline{A}$ has 2 columns and $\underline{C}$ has 3 rows

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Scalar Multiplication

Ex $\underline{A} = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$

$\Rightarrow \quad 4\underline{A} = \begin{pmatrix} 8 & 12 \\ 4 & 16 \end{pmatrix}$

$\Rightarrow \quad -\frac{1}{2}\underline{A} = \begin{pmatrix} -1 & -\frac{3}{2} \\ -\frac{1}{2} & -2 \end{pmatrix}$

Exercise 6B

1)

A	B	C	D	E
2×2	1×2	1×3	3×2	2×3

a) $\underline{B}\,\underline{A}_{1\times2}$ b) $DE_{3\times3}$ c) $CD_{1\times2}$

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3) $\underline{A} = \begin{pmatrix} -1 & -2 \\ 0 & 3 \end{pmatrix}$ $\underline{B} \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

$$\underline{A}\,\underline{B} = \begin{array}{c|ccc} & C_1 & C_2 & C_3 \\ \hline R_1 & -3 & -2 & -1 \\ R_2 & 3 & 3 & 0 \end{array}_{2\times3}$$

$\underline{A}^2 = \begin{pmatrix} 1 & -4 \\ 0 & 9 \end{pmatrix}$

5) $\begin{pmatrix} 2 & a \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 3 & 0 \\ 0 & -1 & 2 \end{pmatrix}$

$$\begin{array}{c|c} & \begin{matrix} c_1 & c_2 & c_3 \end{matrix} \\ \begin{matrix} R_1 \\ R_2 \end{matrix} & \begin{pmatrix} 2 & 6-a & 2a \\ 1 & 4 & -2 \end{pmatrix} \end{array}$$

Identity Matrices

All identity matrices are square

$\underline{I_2} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\quad$ $\underline{I_3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Suppose $\underline{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$\Rightarrow \underline{A}\,\underline{I_2} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$\underline{I_2}\,\underline{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

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15) $\underline{A} = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ $\quad$ $\underline{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\underline{A}^2 = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 4 \\ 6 & 7 \end{pmatrix}$

$$2\underline{A} + 5\underline{I} = 2\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix} + 5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 4 \\ 6 & 2 \end{pmatrix} + \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} 7 & 4 \\ 6 & 7 \end{pmatrix}$$

$$\therefore \quad 2\underline{A} + 5\underline{I} = \underline{A}^2$$

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17)

$$\underline{A} = \begin{pmatrix} 1 & -1 & b \\ a & 2 & 0 \\ 1 & 0 & 3 \end{pmatrix} \qquad \underline{A}^2 = \begin{pmatrix} -4 & -3 & -8 \\ a & 1 & -6 \\ 4 & -1 & 7 \end{pmatrix}$$

Find a, b

$$\underline{A}^2 \begin{pmatrix} & & \\ & -a+4 & \\ & & b+a \end{pmatrix}$$

$b + a = 7$

$\underline{b = -2}$

$-a + 4 = 1$

$\underline{3 = a}$