## edexcel

If the lines meet, $-1+3\lambda = -4+3\mu$ and $2+4\lambda = 2\mu$ Solve to give $\lambda = 0$ ( $\mu = 1$ but this need not be seen). Also $1-\lambda = \alpha$ and so $\alpha = 1$ . $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 3 & 4 \\ 0 & 3 & 2 \end{vmatrix} = -6\mathbf{i}+2\mathbf{j}-3\mathbf{k}$ is perpendicular to both lines and hence to the plane The plane has equation $\mathbf{r.n}=\mathbf{a.n}$ , which is $-6x + 2y - 3z = -14$ ,	M1 M1 A1 B1 (4) M1 A1
Also $1 - \lambda = \alpha$ and so $\alpha = 1$ . $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 3 & 4 \\ 0 & 3 & 2 \end{vmatrix} = -6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ is perpendicular to both lines and hence to the plane	B1 (4) M1 A1
) $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 3 & 4 \\ 0 & 3 & 2 \end{vmatrix}$ = -6 $\mathbf{i}$ +2 $\mathbf{j}$ -3 $\mathbf{k}$ is perpendicular to both lines and hence to the plane	(4) M1 A1
	M1 A1
The plane has equation <b>r.n=a.n</b> , which is $-6x + 2y - 3z = -14$ ,	N/1
i.e. $-6x + 2y - 3z + 14 = 0$ .	A1 o.a.e. (4)
Alternative scheme	
Use (1, -1,2) and ( $\alpha$ , -4,0) in equation $ax+by+cz+d=0$	M1
And third point so three equations, and attempt to solve	M1
Obtain $6x - 2y + 3z =$	A1
(6x - 2y + 3z) - 14 = 0	A1 o.a.e. (4)
$(a_1 - a_2) = i - 3j - 2k$	M1
Use formula $\frac{(\mathbf{a}_1 - \mathbf{a}_2) \bullet \mathbf{n}}{ \mathbf{n} } = \frac{(\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}) \bullet (-6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})}{\sqrt{(36 + 4 + 9)}} = \left(\frac{-6}{7}\right)$	M1
Distance is $\frac{6}{7}$	A1 (3) [11]
	Alternative scheme Use (1, -1,2) and ( $\alpha$ , -4,0) in equation $ax+by+cz+d=0$ And third point so three equations, and attempt to solve Obtain $6x-2y+3z =$ (6x-2y+3z)-14=0 ( $a_1-a_2$ ) = i - 3j - 2k Use formula $\frac{(a_1-a_2) \cdot n}{ n } = \frac{(i-3j-2k) \cdot (-6i+2j-3k)}{\sqrt{(36+4+9)}} = \left(\frac{-6}{7}\right)$

Question Number	Scheme	Marks
7(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 1 & 0 \\ 6 & -2 & 1 \end{vmatrix} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} = 5$ $\mathbf{r} \bullet \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = 5$	M1 A2(1,0) M1A1 (5)
(b)	(2) Equation of $l$ is $\mathbf{r} = \begin{pmatrix} 6\\13\\5 \end{pmatrix} + t \begin{pmatrix} 1\\4\\2 \end{pmatrix}$ At intersection $\begin{pmatrix} 6+t\\13+4t\\5+2t \end{pmatrix} \cdot \begin{pmatrix} 1\\4\\2 \end{pmatrix} = 5$	M1 M1
(c)	(5+2t) (2) $\Rightarrow 6+t+4(13+4t)+2(5+2t)=5 \Rightarrow t=-3$ N is $(3,1,-1)$ $\overrightarrow{PN} \cdot \overrightarrow{PR} = (-3\mathbf{i}-12\mathbf{j}-6\mathbf{k}) \cdot (-5\mathbf{i}-13\mathbf{j}-3\mathbf{k}) = 189$	M1 A1 (4) M1 A1ft
	$\sqrt{9+144+36}\sqrt{25+169+9}\cos NPR = 189$ $NX = NP\sin NPR = \sqrt{189}\sin NPR = 3.61$	A1 M1A1 (5) 14



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Question Number	Scheme	Marks	
6. (a)	$\mathbf{n} = (2\mathbf{j} \cdot \mathbf{k}) \times (3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) = 6\mathbf{i} - 3\mathbf{j} - 6\mathbf{k}$ o.a.e. (e.g. $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ )	M1 A1	(2)
(b)	Line <i>l</i> has direction $2\mathbf{i} - 2\mathbf{j} - \mathbf{k}$ Angle between line <i>l</i> and normal is given by $(\cos\beta \text{ or } \sin\alpha) = \frac{4+2+2}{\sqrt{9}\sqrt{9}} = \frac{8}{9}$	B1 M1 A1ft	
	$\alpha = 90 - \beta = 63$ degrees to nearest degree.	A1 awrt	(4)
(c) Alt 1	Plane <i>P</i> has equation $\mathbf{r}.(2\mathbf{i} - \mathbf{j} - 2\mathbf{k}) = 1$ Perpendicular distance is $\frac{1 - (-7)}{\sqrt{9}} = \frac{8}{3}$	M1 A1 M1 A1	(4)
(c) Alt 2	Parallel plane through A has equation $\mathbf{r} \cdot \frac{2\mathbf{i} - \mathbf{j} - 2\mathbf{k}}{3} = \frac{-7}{3}$	M1 A1 M1	10
	Plane P has equation $\mathbf{r} \cdot \frac{2\mathbf{i} - \mathbf{j} - 2\mathbf{k}}{3} = \frac{1}{3}$ So O lies between the two and perpendicular distance is $\frac{1}{3} + \frac{7}{3} = \frac{8}{3}$	A1	(4)
(c) Alt 3	Distance A to $(3,1,2) = \sqrt{2^2 + 2^2 + 1^2} = 3$ Perpendicular distance is '3' sin $\alpha = 3 \times \frac{8}{9} = \frac{8}{3}$	M1A1 M1A1	
(c) Alt 4	Finding Cartesian equation of plane P: $2x - y - 2z - 1 = 0$ $d = \frac{ n_1 \alpha + n_2 \beta + n_3 \gamma + d }{\sqrt{n_1^2 + n_2^2 + n_3^2}} = \frac{ 2(1) - 1(3) - 2(3) - 1 }{\sqrt{2^2 + 1^2 + 2^2}} = \frac{8}{3}$	M1 A1 M1A1	(4) (4)
A1 (b) B1 M1 1A1ft 2A1 (c) 1M1 1A1 2M1	Angle between $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ and $2\mathbf{i} - 2\mathbf{j} - \mathbf{k}$ , formula of correct form		(+)
2A1	CAO		

Question Number	Scheme	Marks
<b>3.</b> (a)	$AC = 3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}, \qquad BC = -3\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ $AC \times BC = 10\mathbf{i} - 15\mathbf{j} + 30\mathbf{k}$	B1, B1 M1 A1 (4)
(b)	Area of triangle $ABC = \frac{1}{2}  10\mathbf{i} - 15\mathbf{j} + 30\mathbf{k}  = \frac{1}{2}\sqrt{1225} = 17.5$	M1 A1 (2)
(c)	Equation of plane is $10x - 15y + 30z = -20$ or $2x - 3y + 6z = -4$ So <b>r.</b> $(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}) = -4$ or correct multiple	M1 A1 (2) ( 8 marks)

## Notes

**Notes** a1B1:  $AC = 3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$  cao, any form

a2B1:  $BC = -3\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$  cao, any form

a1M1: Attempt to find cross product, modulus of one term correct.

a1A1: cao, any form.

b1M1: modulus of their answer to (a) – condone missing <sup>1</sup>/<sub>2</sub> here. To finding area of triangle by correct method.

b1A1: cao.

c1M1: [Using their answer to (a) to] find equation of plane. Look for a.n or b.n or c.n for p. c1A1: cao