Algebraic Proof

Need an integer say n

even integer	$2n$	where n is an integer
odd integer	$2n+1$	— " —

Prove odd + odd $\rightarrow$ even

Let $2n+1$ be any odd number
$2m+1$ be any odd number

$$2m+1+2n+1 = 2m+2n+2$$
$$= 2(m+n+1)$$

which is even since 2 is a factor

Exercise

Prove odd $\times$ odd $\rightarrow$ odd

As above let odd numbers be
$2n+1$ and $2m+1$

$$(2n+1)(2m+1)$$

$$= 4nm+2m+2n+1$$
$$= 2(2nm+m+n)+1$$

which is odd because 2 is not a factor

Prove the sum of 3 consecutive integers is divisible by 3

Let numbers be $n, n+1, n+2$

$$\text{Sum} = n + n+1 + n+2$$
$$= 3n + 3$$
$$= 3(n+1)$$

Divisible by 3 since 3 is a factor

Sequences

Linear Sequences

Ex1 $\quad 4 \overset{3}{\quad} 7 \overset{6}{\quad} 10 \overset{9}{\quad} 13 \overset{12}{\quad} \ldots$

Common difference = 3

$$n^{th} \text{ term} = 3n + 1$$

Ex2 $\quad 2 \overset{7}{\quad} 9 \overset{14}{\quad} 16 \overset{21}{\quad} 23 \overset{28}{\quad} \ldots$

common diff = 7

$$n^{th} \text{ term} = 7n - 5$$

Ex3 58, 55, 52, 49

common diff $= -3$

n^{th} term $61 - 3n$

Quadratic Sequences

Consider

2nd diff		2	2	2	2	
1st diff	4	6	8	10	12	
	7	11	17	25	35	47
n^2	1	4	9	16	25	36
	6	7	8	9	10	11
$+n$	1	2	3	4	5	6
$+5$	5	5	5	5	5	5

n^{th} term $= n^2 + n + 5$

Ex2

2nd diff		4	4	4	4	
1st diff	3	7	11	15	19	
	6	9	16	27	42	61
$2n^2$	2	8	18	32	50	72
	4	1	-2	-5	-8	-11
$-3n$	-3	-6	-9	-12	-15	-18

+7	7	7	7	7	7	7

n^{th} term $= 2n^2 - 3n + 7$

Exercise

2nd		6		6		6			
1st	4		10		16		22		
	10		14		24		40		62
$3n^2$	3		12		27		48		75
	7		2		-3		-8		-13
$-5n$	-5		-10		-15		-20		-25
+12	12		12		12		12		12

n^{th} term $= 3n^2 - 5n + 12$

2nd		2		2		2			
1st	1		3		5		7		
	19		20		23		28		35
n^2	1		4		9		16		25
	18		16		14		12		10
$-2n$	-2		-4		-6		-8		-10
+20	20		20		20		20		20

n^{th} term $= n^2 - 2n + 20$