Higher Derivatives Ex 2B

1) a) $f(x) = e^{2x}$

$$f'(x) = 2e^{2x}$$
$$f''(x) = 4e^{2x}$$
$$f'''(x) = 8e^{2x}$$
$$f^{(n)}(x) = 2^n e^{2x}$$

b) $f(x) = (1+x)^n$

$$f'(x) = n(1+x)^{n-1}$$
$$f''(x) = n(n-1)(1+x)^{n-2}$$
$$f'''(x) = n(n-1)(n-2)(1+x)^{n-3}$$
$$f^{(n)}(x) = n!$$

c) $f(x) = xe^x$

$$f'(x) = xe^x + e^x$$
$$f''(x) = xe^x + e^x + e^x$$
$$= xe^x + 2e^x$$
$$f'''(x) = xe^x + 3e^x$$
$$f^{(n)}(x) = xe^x + ne^x$$

d) $f(x) = \ln(1+x)$

$$f'(x) = \frac{1}{1+x} = (1+x)^{-1}$$
$$f''(x) = -1(1+x)^{-2} = -\frac{1}{(1+x)^2}$$
$$f'''(x) = 2(1+x)^{-3}$$

$$f^{iv}(x) = -6(1+x)^{-4}$$

$$f^{(5)}(x) = 24(1+x)^{-5}$$

$$f^{(n)}(x) = (-1)^{n+1}(n-1)!(1+x)^{-n}$$

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3) $y = \sin^2 3x = (\sin 3x)^2$

a) $\dfrac{dy}{dx} = 2(\sin 3x)^1 \times 3\cos 3x = 3(2\sin 3x \cos 3x)$

$= 3\sin 6x$

b) $\dfrac{d^2y}{dx} = 18\cos 6x$

$\dfrac{d^3y}{dx^3} = -108\sin 6x$

$\dfrac{d^4y}{dx^4} = -648\cos 6x$

c) when $x = \frac{\pi}{6}$ $\quad \dfrac{d^4y}{dx^4} = -648\cos\pi$

$= 648$

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5)

$$y = \sec x = \frac{1}{\cos x} = (\cos x)^{-1}$$

$$\frac{dy}{dx} = -1(\cos x)^{-2}(-\sin x) = \frac{\sin x}{\cos^2 x}$$

$$= \sin x(\cos x)^{-2}$$

$$\frac{d^2y}{dx^2} = \sin x \times -2(\cos x)^{-3}(-\sin x) + (\cos x)^{-2}\cos x$$

$$= \frac{2\sin^2 x}{\cos^3 x} + \frac{\cos x}{\cos^2 x}$$

$$= 2\sec^3 x \sin^2 x + \frac{1}{\cos x}$$

$$= 2\tan^2 x \sec x + \sec x$$

$$= 2(\sec^2 x - 1)\sec x + \sec x$$

$$= 2\sec^3 x - 2\sec x + \sec x$$

$$= 2\sec^3 x - \sec x$$
