Completing the Square

Consider $(x+a)^2$

$$= (x+a)(x+a)$$

$$= x^2 + ax + ax + a^2$$

$$= x^2 + 2ax + a^2$$

When completing the square we wish to write a quadratic expression as $(x+a)^2 + b$

Ex1

$$x^2 + 6x + 10$$

$$= (x+3)^2 + 10 - 9$$

$$= (x+3)^2 + 1$$

$(x+3)(x+3)$

$= x^2 + 3x + 3x + 9$

$x^2 + 6x + 9$

Writing $(x+3)^2$ introduced $+9$ we did not want so we subtracted it. The $+3$ came from half the $+6$

Ex2

$$x^2 + 8x + 5$$

$$= (x+4)^2 + 5 - 16$$

$$= (x+4)^2 - 11$$

Ex3

$$x^2 - 10x + 50$$

$$= (x-5)^2 + 50 - 25$$

$$= (x-5)^2 + 25$$

Exercise

1) $x^2 + 2x + 3$

$= (x+1)^2 + 3 - 1$

$= (x+1)^2 + 2$

2) $x^2 + 10x + 30$

$= (x+5)^2 + 30 - 25$

$= (x+5)^2 + 5$

3) $x^2 - 4x + 10$

$= (x-2)^2 + 10 - 4$

$= (x-2)^2 + 6$

4) $x^2 - 8x - 3$

$= (x-4)^2 - 3 - 16$

$= (x-4)^2 - 19$

5) $x^2 + 5x + 10$

$= \left(x + \frac{5}{2}\right)^2 + 10 - \frac{25}{4}$

$= \left(x + \frac{5}{2}\right)^2 + \frac{40}{4} - \frac{25}{4}$

$= \left(x + \frac{5}{2}\right)^2 + \frac{15}{4}$

6) $x^2 - 3x + 5$

$= \left(x - \frac{3}{2}\right)^2 + 5 - \frac{9}{4}$

$= \left(x - \frac{3}{2}\right)^2 + \frac{20}{4} - \frac{9}{4} \quad = \left(x - \frac{3}{2}\right)^2 + \frac{11}{4}$

7) $x^2 + x + 3$

$= \left(x + \frac{1}{2}\right)^2 + 3 - \frac{1}{4}$

$= \left(x + \frac{1}{2}\right)^2 + \frac{12}{4} - \frac{1}{4} \quad = \left(x + \frac{1}{2}\right)^2 + \frac{11}{4}$