

U61M

Year 13 A-Level Mathematics

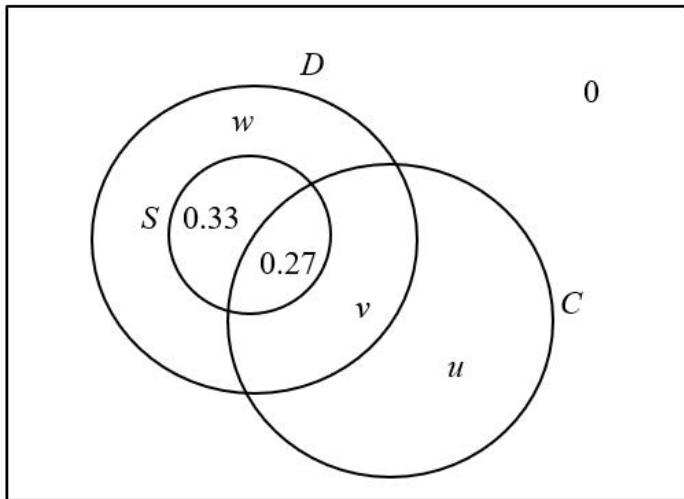
Mock Applied Paper 48 Marks

1 Hour

January 2019

1. The Venn diagram shows the probabilities of students' lunch boxes containing a drink, sandwiches and a chocolate bar.

D is the event that a lunch box contains a drink,
 S is the event that a lunch box contains sandwiches,
 C is the event that a lunch box contains a chocolate bar,
 u, v and w are probabilities.



- (a) Write down $P(S \cap D')$ (1)

One day, 80 students each bring in a lunch box.

Given that all 80 lunch boxes contain sandwiches and a drink,

- (b) estimate how many of these 80 lunch boxes will contain a chocolate bar. (3)

Given that the events S and C are independent and that $P(D | C) = \frac{14}{15}$

- (c) calculate the value of u , the value of v and the value of w .

a) $P(S \cap D') = 0$ (7)

b) $P(C \setminus (S \cap D)) = \frac{P(C \cap (S \cap D))}{P(S \cap D)}$

$$= \frac{0.27}{(0.27+0.33)} = \frac{0.27}{0.6} = 0.45 \quad \text{Estimate for boxes containing choc}$$

$$= 80 \times 0.45 = 36$$

c) $P(D | C) = \frac{14}{15} = \frac{P(D \cap C)}{P(C)}$

Question 1 continued

$$\frac{14}{15} = \frac{0.27+v}{0.27+v+u}$$
(1)

s and c independent so

$$P(s \cap c) = P(s) \times P(c)$$

$$0.27 = 0.6 \times (v+u+0.27)$$

$$\frac{0.27}{0.6} = v+u+0.27$$

$$0.45 - 0.27 = v+u$$

$$0.18 = v+u$$
(2)

Sub for $v+u$ in (1)

$$\frac{14}{15} = \frac{0.27+v}{0.27+0.18}$$

$$\frac{14}{15} = \frac{0.27+v}{0.45}$$

$$\frac{14}{15} \times 0.45 - 0.27 = v$$

$$\underline{v = 0.15}$$

From (2)

$$v = 0.18 - u$$

$$u = 0.18 - 0.15$$

$$\underline{u = 0.03}$$

$$w = 1 - (0.6 + v + u) = 1 - (0.6 + 0.18) = 0.22$$

$$u = 0.03$$

$$v = 0.15$$

$$w = 0.22$$

(Total for Question 1 is 11 marks)

2. The lifetimes of batteries sold by company X are normally distributed, with mean 150 hours and standard deviation 25 hours.

A box contains 12 batteries from company X .

- (a) Find the expected number of these batteries that have a lifetime of more than 160 hours. (3)

The lifetimes of batteries sold by company Y are normally distributed, with mean 160 hours and 80% of these batteries have a lifetime of less than 180 hours.

- (b) Find the standard deviation of the lifetimes of batteries from company Y . (3)

Both companies sell their batteries for the same price.

- (c) State which company you would recommend. Give reasons for your answer.

a) $X \sim N(\mu, \sigma^2)$ $P(X > 160) = 0.34458$ (by calc) (2)

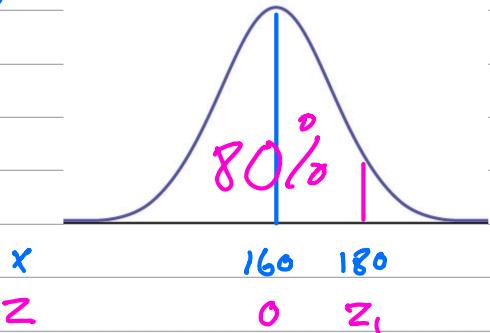
Expected number lasting more than 160 hours

$$= 12 \times 0.34458 = 4.13496$$

$$= 4.13$$

to 3 s.f.

b)



$$Z_1 = \Phi^{-1}(0.8)$$

$$Z_1 = 0.841620847$$

$$Z = \frac{x - \mu}{\sigma}$$

$$\sigma z = x - \mu$$

$$\sigma = \frac{x - \mu}{z}$$

$$\sigma = \frac{180 - 160}{0.841620847}$$

$$\sigma = 23.76 = 23.8 \text{ to 3sf.}$$

Question 2 continued

c) Recommend Company Y

Standard deviations are similar but Y

batteries have a greater mean life than

X batteries

(Total for Question 2 is 8 marks)

SECTION B: MECHANICS

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

[In this question position vectors are given relative to a fixed origin O .]

3. A particle, P , moves with constant acceleration $(\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-2}$.

At time $t = 0$ seconds, the particle is at the point A with position vector $(2\mathbf{i} + 5\mathbf{j}) \text{ m}$ and is moving with velocity $\mathbf{u} \text{ m s}^{-1}$.

At time $t = 3$ seconds, P is at the point B with position vector $(-2.5\mathbf{i} + 8\mathbf{j}) \text{ m}$.

Find \mathbf{u} .

(4)

$$\underline{s} - \underline{s}_0 = \underline{u}t + \frac{1}{2}\underline{a}t^2$$

$$\underline{s} - \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 1 \\ -2 \end{pmatrix} t^2$$

$$t = 3 \\ \underline{s} = \begin{pmatrix} -2.5 \\ 8 \end{pmatrix} \quad \begin{pmatrix} -2.5 \\ 8 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}(3) + \frac{1}{2} \begin{pmatrix} 1 \\ -2 \end{pmatrix}(3)^2$$

$$\begin{pmatrix} -4.5 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \end{pmatrix} = 3 \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\begin{pmatrix} -9 \\ 12 \end{pmatrix} = 3 \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\underline{u} = -3\mathbf{i} + 4\mathbf{j} \quad \text{ms}^{-1}$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

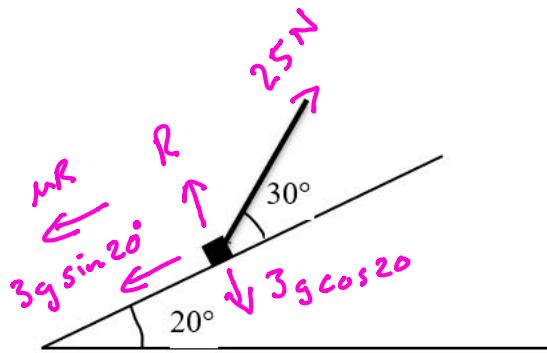
DO NOT WRITE IN THIS AREA

Question 3 continued

(Total for Question 3 is 4 marks)

4.

$$\mu = 0.3$$

**Figure 1**

A small box of mass 3 kg moves on a rough plane which is inclined at an angle of 20° to the horizontal.

The box is pulled up a line of greatest slope of the plane using a rope which is attached to the box.

The rope makes an angle of 30° with the plane, as shown in Figure 1.

The rope lies in the vertical plane which contains a line of greatest slope of the plane.

The coefficient of friction between the box and the plane is 0.3.

The tension in the rope is 25 N.

The box is modelled as a particle, the rope is modelled as a light inextensible string and air resistance is ignored.

(a) Using the model, find the acceleration of the box.

(7)

(b) Suggest one improvement to the model that would make it more realistic.

(1)

The rope now breaks and the box slows down and comes to rest.

(c) Show that, after the box comes to rest, it immediately starts to move down the plane.

(3)

a) \perp to slope $R + 25 \sin 30 = 3g \cos 20$

$$R = 3g \cos 20 - 25 \sin 30$$

\parallel to slope $F = ma$

$$25 \cos 30 - 3g \sin 20 - \mu R = 3a$$

Question 4 continued

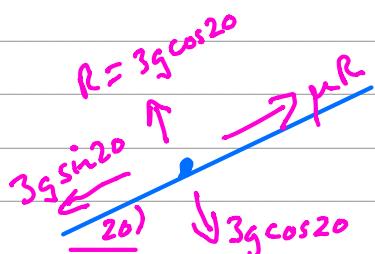
$$a = \frac{25\cos 30 - 3g\sin 20 - 0.3(3g\cos 20 - 25\sin 30)}{3}$$

$$a = 2.3524$$

$$a = 2.35 \text{ m s}^{-2}$$

b) Consider air resistance

c)



$$3g \sin 20 = 10.06 \text{ N}$$

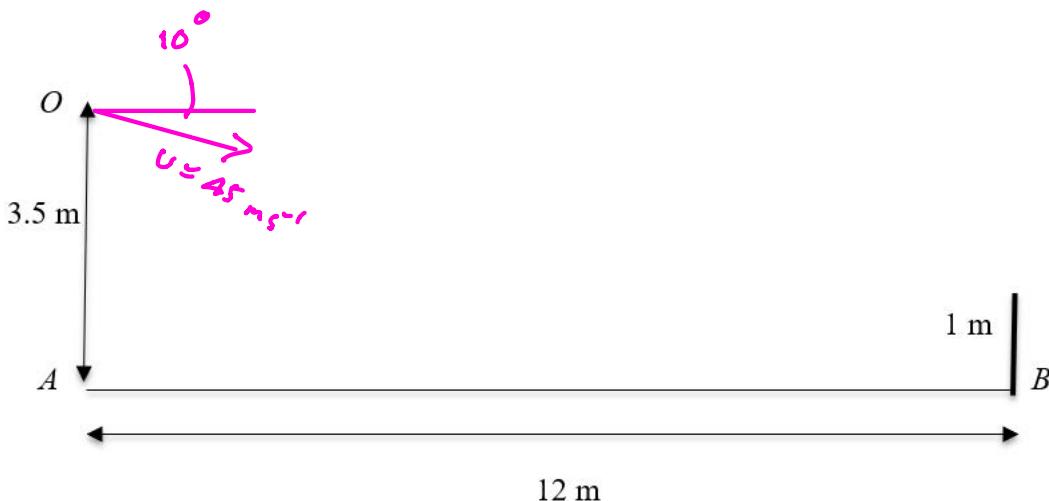
$$\mu R = 0.3 \times 3g \cos 20^\circ = 8.29 \text{ N}$$

$$\text{Resultant force down slope} = 10.06 - 8.29 = 1.77 \text{ N}$$

\therefore immediately accelerates down slope

(Total for Question 4 is 11 marks)

5.

**Figure 3**

A tennis player serves a ball so as to pass over the net.

The ball is given an initial velocity of 45 m s^{-1} in a direction 10° below the horizontal.

The ball is struck at a point O which is 3.5 m vertically above the point A which is on horizontal ground.

The bottom of the net is the point B which is on the ground and $AB = 12 \text{ m}$.

The height of the net is 1 m, as shown in Figure 3.

The ball is modelled as a particle moving freely under gravity.

The ball passes over the net at a point which is vertically above B.

Using the model,

(a) find, in centimetres to 2 significant figures, the distance between the ball and the top of the net, as the ball passes over the net,

(8)

(b) find, to 2 significant figures, the speed of the ball as it passes over the net.

(4)

(c) State two limitations of the model that could affect the reliability of your answers.

(2)

$$\alpha) \quad u_x = 45 \cos 10^\circ \quad u_y = -45 \sin 10^\circ$$

Horizontal motion time to reach net $t = \frac{12}{45 \cos 10^\circ}$

$$t = 0.2707804$$

Vertical motion $y - y_0 = ut + \frac{1}{2}at^2$

$$y - 3.5 = -45 \sin 10^\circ \times 0.2707804 - 4.9 \times 0.2707804^2$$

Question 5 continued

$$y = 1.0248 \text{ m}$$

so 2.5 cm above net to 2 s.f.

b) $v_x = u_x = 45 \cos 10 = 44.3163 \text{ ms}^{-1}$

$$\begin{aligned} v_y &= u_y + at \\ &= -45 \sin 10 - 9.8 \times 0.2707804 \\ &= -10.4678 \text{ ms}^{-1} \end{aligned}$$

$$\text{Speed} = \sqrt{44.3163^2 + (-10.4678)^2}$$

$$= 45.5358$$

$$= 46 \text{ ms}^{-1} \text{ to 2 s.f.}$$

c) air resistance, size of ball, spin on ball

Question 5 continued

(Total for Question 5 is 14 marks)

TOTAL FOR PAPER IS 48 MARKS