Translations of Graphs of Functions

Let $y = f(x)$

be a function of x

then $y = f(x-a)$ is translation by $\begin{pmatrix} a \\ 0 \end{pmatrix}$

$y = f(x) + b$ is a translation by $\begin{pmatrix} 0 \\ b \end{pmatrix}$

Examples

If $y = f(x)$

1. $f(x-3)+4$ is a translation by $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$
2. $f(x+1)$ is a translation by $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$
3. $f(x)-7$ is a translation by $\begin{pmatrix} 0 \\ -7 \end{pmatrix}$
4. $f(x+2)+2$ is a translation by $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$
5. $f(x+6)-3$ ——— " ——— by $\begin{pmatrix} -6 \\ -3 \end{pmatrix}$
6. $f(x-4)+1$ ——— " ——— by $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

Completing the square can be used to sketch quadratic graphs

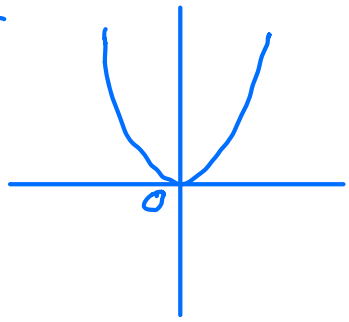
Ex 1

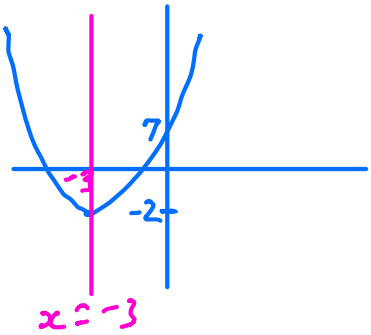
$$y = x^2 + 6x + 7$$

$$y = (x+3)^2 + 7 - 9$$

$$y = (x+3)^2 - 2$$

$y = x^2$





Line of symmetry $x = -3$

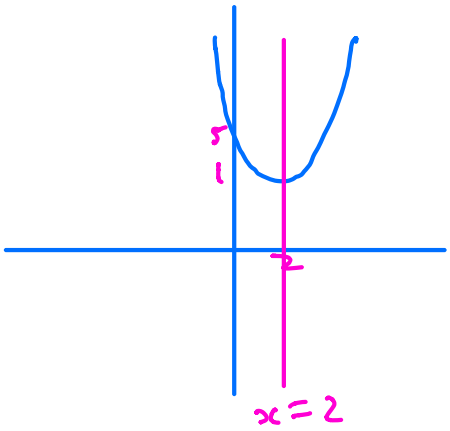
Min point $(-3, -2)$

Ex 2

$$y = x^2 - 4x + 5$$

$$y = (x-2)^2 + 5 - 4$$

$$y = (x-2)^2 + 1$$



Line of symmetry $x = 2$

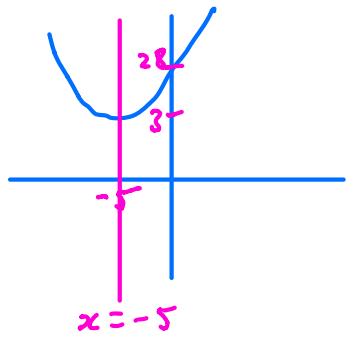
Min point $(2, 1)$

Exercise

1) $y = x^2 + 10x + 28$

$y = (x+5)^2 + 28 - 25$

$y = (x+5)^2 + 3$



Symmetry

$x = -5$

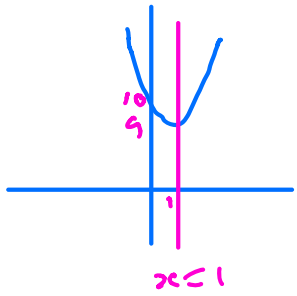
Min point

$(-5, 3)$

2) $y = x^2 - 2x + 10$

$y = (x-1)^2 + 10 - 1$

$y = (x-1)^2 + 9$



Symmetry

$x = 1$

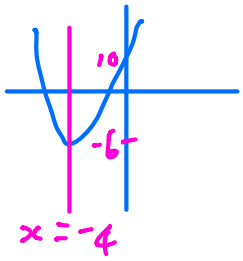
Min point

$(1, 9)$

3) $y = x^2 + 8x + 10$

$y = (x+4)^2 + 10 - 16$

$y = (x+4)^2 - 6$



Symmetry

$x = -4$

Min point

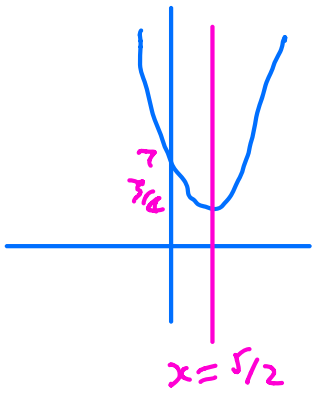
$(-4, -6)$

4) $y = x^2 - 5x + 7$

$y = \left(x - \frac{5}{2}\right)^2 + 7 - \frac{25}{4}$

$y = \left(x - \frac{5}{2}\right)^2 + \frac{28}{4} - \frac{25}{4}$

$y = \left(x - \frac{5}{2}\right)^2 + \frac{3}{4}$



Symmetry $x = \frac{5}{2}$

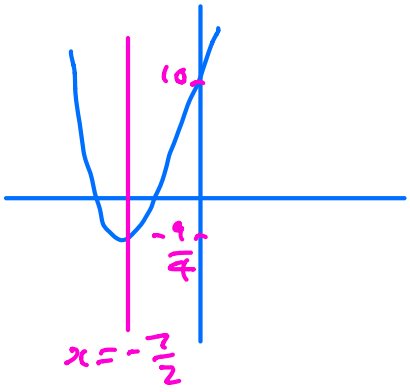
Min point $(5/2, 3/4)$

5) $y = x^2 + 7x + 10$

$y = \left(x + \frac{7}{2}\right)^2 + 10 - \frac{49}{4}$

$y = \left(x + \frac{7}{2}\right)^2 + \frac{40}{4} - \frac{49}{4}$

$y = \left(x + \frac{7}{2}\right)^2 - \frac{9}{4}$



Symmetry line $x = -\frac{7}{2}$

Min point $\left(-\frac{7}{2}, -\frac{9}{4}\right)$