Linear Transformations

All linear transformations map origin $\begin{pmatrix}0\\0\end{pmatrix}$ to origin $\begin{pmatrix}0\\0\end{pmatrix}$

Linear Transformations are of the form

$$\begin{pmatrix}x\\y\end{pmatrix} \longmapsto \begin{pmatrix}ax+by\\cx+dy\end{pmatrix}$$

then $$\begin{pmatrix}a & b\\c & d\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}ax+by\\cx+dy\end{pmatrix}$$

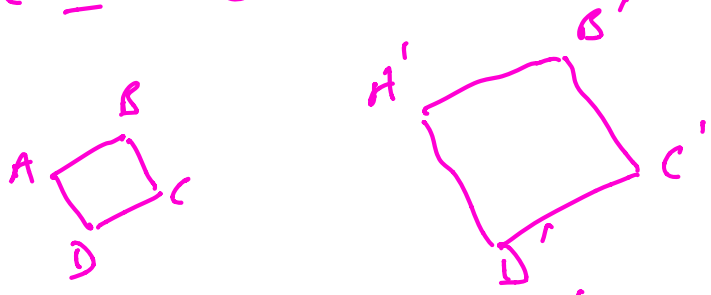
$$\begin{pmatrix}-1 & 2\\2 & 1\end{pmatrix}\begin{array}{c}\begin{array}{cccc}a & b & c & d\end{array}\\ \begin{pmatrix}1 & 3 & 3 & 1\\1 & 1 & 3 & 3\end{pmatrix}\end{array} = \begin{array}{c}\begin{array}{cccc}a' & b' & c' & d'\end{array}\\ \begin{pmatrix}1 & -1 & 3 & 5\\3 & 7 & 9 & 5\end{pmatrix}\end{array}$$

If $\underline{M}$ is a linear transformation matrix

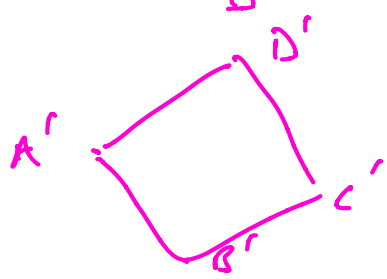
then applying the transformation multiplies areas by $\det \underline{M}$

If $\det \underline{M} > 0$ orientation is preserved

If $\det \underline{M} < 0$ orientation is reversed



preserved

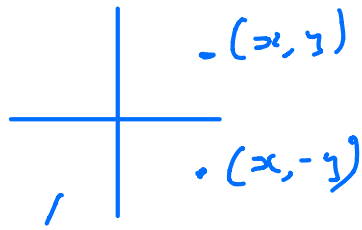


reversed

Exercise 7B

1) Reflection in x-axis $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 1 & 3 & 3 \\ 3 & 3 & 2 \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{ccc} A' & B' & C' \end{array} \\ \begin{pmatrix} 1 & 3 & 3 \\ -3 & -3 & -2 \end{pmatrix} \end{array}$$

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3 a) θ° anti-clockwise $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

90° anti-clockwise $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

270° anti-clockwise $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

45° anti $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

210° anti $\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$

135° clock

θ clock $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

$\begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$

Hwk Ex 78 even numbers
