Functions Example

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6. The function f is defined by

$$\mathrm{f} : x \rightarrow \mathrm{e}^{2x} + k^2, \quad x \in \mathbb{R}, \quad k \text{ is a positive constant.}$$

(a) State the range of f.

(1)

(b) Find f^{-1} and state its domain.

(3)

The function g is defined by

$$\mathrm{g} : x \rightarrow \ln(2x), \quad x > 0$$

(c) Solve the equation

$$\mathrm{g}(x) + \mathrm{g}(x^2) + \mathrm{g}(x^3) = 6$$

giving your answer in its simplest form.

(4)

(d) Find $\mathrm{fg}(x)$, giving your answer in its simplest form.

(2)

(e) Find, in terms of the constant k, the solution of the equation

$$\mathrm{fg}(x) = 2k^2$$

(2)

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Question 6 continued

Question Number	Scheme	Marks
6.(a)	$\mathrm{f}(x) > k^2$	B1
		(1)
(b)	$y = \mathrm{e}^{2x} + k^2 \Rightarrow \mathrm{e}^{2x} = y - k^2$	M1
	$\Rightarrow x = \frac{1}{2}\ln(y - k^2)$	dM1
	$\Rightarrow \mathrm{f}^{-1}(x) = \frac{1}{2}\ln(x - k^2), \quad x > k^2$	A1
		(3)
(c)	$\ln 2x + \ln 2x^2 + \ln 2x^3 = 6$	M1
	$\Rightarrow \ln 8x^6 = 6$	M1
	$\Rightarrow 8x^6 = \mathrm{e}^6 \Rightarrow x = ..$	M1
	$\Rightarrow x = \left(\frac{\mathrm{e}}{\sqrt[6]{8}}\right) = \frac{\mathrm{e}}{\sqrt{2}}$ (Ignore any reference to $-\frac{\mathrm{e}}{\sqrt{2}}$)	A1
		(4)
(d)	$\mathrm{fg}(x) = e^{2\times\ln(2x)} + k^2$	M1
	$\Rightarrow \mathrm{fg}(x) = (2x)^2 + k^2 = 4x^2 + k^2$	A1
		(2)
(e)	$\mathrm{fg}(x) = 2k^2 \Rightarrow 4x^2 + k^2 = 2k^2$	
	$\Rightarrow 4x^2 = k^2 \Rightarrow x = ..$	M1
	$\Rightarrow x = \frac{k}{2}$ **only**	A1
		(2) **12 marks**
(alt c)	$\ln 2x + \ln 2x^2 + \ln 2x^3 = 6$	M1
	$\Rightarrow \ln 2 + \ln x + \ln 2 + 2\ln x + \ln 2 + 3\ln x = 6$	
	$\Rightarrow 3\ln 2 + 6\ln x = 6$	
	$\Rightarrow \ln x = 1 - \frac{1}{2}\ln 2$	M1
	$\Rightarrow x = e^{1-\frac{1}{2}\ln 2}, = \frac{\mathrm{e}}{\sqrt{2}}$ (Ignore any reference to $-\frac{\mathrm{e}}{\sqrt{2}}$)	M1, A1
(alt e)		(4)
	$\Rightarrow 2\ln(2x) = \ln(2k^2 - k^2)$	
	$\Rightarrow \ln(2x)^2 = \ln(k^2), \Rightarrow 4x^2 = k^2 \Rightarrow x = \frac{k}{2}$	M1, A1

(a)

B1 States the correct range for f. Accept $f(x) > k^2, f > k^2, Range > k^2, (k^2, \infty), y > k^2$ Range is greater than k^2

Do not accept $f(x) \geq k^2, x > k^2, [k^2, \infty)$

(b)

M1 Attempts to make x or a swapped y the subject of the formula. Score for $y = e^{2x} + k^2 \Rightarrow e^{2x} = y \pm k^2$ and proceeding to $x = \ln\ldots$ The minimum expectation is that e^{2x} is made the subject before taking ln's

dM1 Dependent upon the previous M having been scored. It is for proceeding by firstly taking ln's of the whole rhs, not the individual elements, and then dividing by 2. Score M1, dM1 for writing down

$x = \frac{1}{2}\ln(y \pm k^2)$ or alternatively $y = \frac{1}{2}\ln(x \pm k^2)$. Condone missing brackets for this mark.

A1 The correct answer in terms of x including the bracket **and** the domain $f^{-1}(x) = \frac{1}{2}\ln(x - k^2), \quad x > k^2$.

Accept equivalent answers like $y = 0.5\ln\left|x - k^2\right|$, Domain greater than k^2, $\left(k^2, \infty\right)$

(c)

M1 Attempts to solve equation by writing down $\ln 2x + \ln 2x^2 + \ln 2x^3 = 6$

M1 Uses addition laws of logs to write in the form $\ln Ax^n = 6$

M1 Takes exp's (correctly) and proceeds to a solution for $x = ..$

A1 Correct solution and correct answer. $x = \frac{e}{\sqrt{2}}$. You may ignore any reference to $x = -\frac{e}{\sqrt{2}}$

Special caseS. Candidate who solve (and treat it as though it was bracketed)

S. Case 1 $\ln 2x + \ln 2x^2 + \ln 2x^3 = 6 \Rightarrow \ln 2x + 2\ln 2x + 3\ln 2x = 6 \Rightarrow 6\ln 2x = 6 \Rightarrow x = \frac{e}{2}$

S. Case 2 $\ln 2x + \ln(2x)^2 + \ln(2x)^3 = 6 \Rightarrow 6\ln 2x = 6 \Rightarrow \ln 2x = 1 \Rightarrow x = \frac{e}{2}$

S. Case 3 $\ln 2x + \ln(2x)^2 + \ln(2x)^3 = 6 \Rightarrow \ln 2x + \ln 4x^2 + \ln 8x^3 = 6 \Rightarrow \ln 64x^6 = 6 \Rightarrow 64x^6 = e^6 \Rightarrow x = \frac{e}{2}$

scores M0 (Incorrect statement/ may be implied by subsequent work), M1 (Correct ln laws), M1 (Correct method of arriving at x=) , A0

(d) For the purposes of marking you can score (d) and (e) together

M1 Correct order of applying g before f to give a correct unsimplified answer. Accept $y =$

Accept versions of $fg(x) = e^{2\times\ln(2x)} + k^2, y = e^{\ln(2x)^2} + k^2$

A1 A correct simplified answer $fg(x) = (2x)^2 + k^2$, or $fg(x) = 4x^2 + k^2$. Accept $y =$

(e)

M1 Sets the answer to (d) in the form $Ax^2 + k^2 = 2k^2$, where $A = 2$ or 4 and proceeds in the correct order to reach an equation of the form $Ax^2 = k^2$.

In the alternative method it would be for reaching $\ln(Ax^2) = \ln(k^2)$, $A = 2$ or 4 or any equivalent form ln…= ln…

A1 $x = \frac{k}{2}$ **only.** The answer $x = \pm\frac{k}{2}$ is A0.