Vectors - 2D SUVAT

2 A particle P moves with acceleration $(-3\mathbf{i} + 12\mathbf{j})\,\mathrm{m\,s^{-2}}$. Initially the velocity of P is $4\mathbf{i}\,\mathrm{m\,s^{-1}}$.

(a) Find the velocity of P at time t seconds. *(2 marks)*

(b) Find the speed of P when $t = 0.5$. *(3 marks)*

6. A particle P moves with constant acceleration $(2\mathbf{i} - 5\mathbf{j})$ m s^{-2}. At time $t = 0$, P has speed u m s^{-1}. At time $t = 3$ s, P has velocity $(-6\mathbf{i} + \mathbf{j})$ m s^{-1}.

Find the value of u.

(Total 5 marks)

6 The points A and B have position vectors $(3\mathbf{i} + 2\mathbf{j})$ metres and $(6\mathbf{i} - 4\mathbf{j})$ metres respectively. The vectors $\mathbf{i}$ and $\mathbf{j}$ are in a horizontal plane.

(a) A particle moves from A to B with constant velocity $(\mathbf{i} - 2\mathbf{j})\,\mathrm{m\,s^{-1}}$. Calculate the time that the particle takes to move from A to B. *(3 marks)*

(b) The particle then moves from B to a point C with a constant acceleration of $2\mathbf{j}\,\mathrm{m\,s^{-2}}$. It takes 4 seconds to move from B to C.

(i) Find the position vector of C. *(4 marks)*

(ii) Find the distance AC. *(2 marks)*

8 A particle is initially at the origin, where it has velocity $(5\mathbf{i} - 2\mathbf{j})\,\mathrm{m\,s^{-1}}$. It moves with a constant acceleration $\mathbf{a}\,\mathrm{m\,s^{-2}}$ for 10 seconds to the point with position vector $75\mathbf{i}$ metres.

(a) Show that $\mathbf{a} = 0.5\mathbf{i} + 0.4\mathbf{j}$. *(3 marks)*

(b) Find the position vector of the particle 8 seconds after it has left the origin. *(3 marks)*

(c) Find the position vector of the particle when it is travelling parallel to the unit vector $\mathbf{i}$. *(6 marks)*

8 A boat is initially at the origin, heading due east at $5\,\text{m}\,\text{s}^{-1}$. It then experiences a constant acceleration of $(-0.2\mathbf{i} + 0.25\mathbf{j})\,\text{m}\,\text{s}^{-2}$. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

(a) State the initial velocity of the boat as a vector. *(1 mark)*

(b) Find an expression for the velocity of the boat t seconds after it has started to accelerate. *(2 marks)*

(c) Find the value of t when the boat is travelling due north. *(3 marks)*

(d) Find the bearing of the boat from the origin when the boat is travelling due north. *(6 marks)*

8. [*In this question, the unit vectors* **i** *and* **j** *are horizontal vectors due east and north respectively.*]

At time $t = 0$, a football player kicks a ball from the point A with position vector $(2\mathbf{i} + \mathbf{j})$ m on a horizontal football field. The motion of the ball is modelled as that of a particle moving horizontally with constant velocity $(5\mathbf{i} + 8\mathbf{j})$ m s^{-1}. Find

(a) the speed of the ball,

(2)

(b) the position vector of the ball after t seconds.

(2)

The point B on the field has position vector $(10\mathbf{i} + 7\mathbf{j})$ m.

(c) Find the time when the ball is due north of B.

(2)

At time $t = 0$, another player starts running due north from B and moves with constant speed v m s^{-1}. Given that he intercepts the ball,

(d) find the value of v.

(6)

(e) State one physical factor, other than air resistance, which would be needed in a refinement of the model of the ball's motion to make the model more realistic.

(1)

6. [*In this question the horizontal unit vectors* **i** *and* **j** *are due east and due north respectively*.]

A model boat A moves on a lake with constant velocity $(-\mathbf{i} + 6\mathbf{j})$ m s^{-1}. At time $t = 0$, A is at the point with position vector $(2\mathbf{i} - 10\mathbf{j})$ m. Find

(a) the speed of A,

(2)

(b) the direction in which A is moving, giving your answer as a bearing.

(3)

At time $t = 0$, a second boat B is at the point with position vector $(-26\mathbf{i} + 4\mathbf{j})$ m.

Given that the velocity of B is $(3\mathbf{i} + 4\mathbf{j})$ m s^{-1},

(c) show that A and B will collide at a point P and find the position vector of P.

(5)

Given instead that B has speed 8 m s^{-1} and moves in the direction of the vector $(3\mathbf{i} + 4\mathbf{j})$,

(d) find the distance of B from P when $t = 7$ s.

(6)