Cones, Spheres, Frustums, Pyramids
Cone

$r=$ radius
$h=$ vertical height
$l=$ slope length
Pythagoras applies $r^{2}+h^{2}=l^{2}$

$$
\begin{gathered}
\text { Volume }=\frac{1}{3} \pi r^{2} h \\
\text { Curved Surface Area }=\pi r l
\end{gathered}
$$

Formulae for cone and sphere are given on exam paper.

Sphere


$$
\begin{aligned}
& \text { Volume }=\frac{4}{3} \pi r^{3} \\
& \text { Surface Area }=4 \pi r^{2}
\end{aligned}
$$

Frustum


A cone with the top chopped off parallel to the base

Volume and surface area are found by considering the original cone and the cone removed


Pyramid


$$
\begin{aligned}
\text { Volume }= & \frac{1}{3} \times \text { area of base } \times \text { height } \\
\text { Surface Area }= & \text { Area of Base } \\
& + \text { Area of } n \text { Triangles }
\end{aligned}
$$

for $n$ sided base

Rearranging Formulae
Sphere

$$
\begin{aligned}
V & =\frac{4}{3} \pi r^{3} \\
3 V & =4 \pi r^{3} \\
\frac{3 V}{4 \pi} & =r^{3} \\
\sqrt[3]{\frac{3 V}{4 \pi}} & =r
\end{aligned}
$$

Examples

1) Find volume of
2) Find radios of sphere radius 5 cm sphere with volume $1000 \mathrm{~cm}^{3}$

$$
\begin{aligned}
V & =\frac{4}{3} \pi r^{3} \\
V & =\frac{4}{3} \pi \times 5^{3} \\
& =524 \mathrm{~cm}^{3}
\end{aligned}
$$

$$
\begin{aligned}
& r=\sqrt[3]{\frac{3 V}{4 \pi}} \\
& r=\sqrt[3]{\frac{3 \times 1000}{4 \pi}} \\
& r=6.20 \mathrm{~cm}
\end{aligned}
$$

3) Find the surface area of a sphere with volume $5000 \mathrm{~cm}^{3}$

$$
\begin{aligned}
r=\sqrt[3]{\frac{3 v}{4 \pi}} & =\sqrt[3]{\frac{3 \times 5000}{4 \pi}}=10.6078 \mathrm{~cm} \\
\text { Surface Area } & =4 \pi r^{2} \\
& =4 \pi \times 10.6078^{2} \\
& =1414 \mathrm{~cm}^{2}
\end{aligned}
$$

4) Find the volume and total surface area of a solid cone radius 5 cm and height 12 cm


$$
\begin{aligned}
l^{2} & =5^{2}+12^{2} \\
& =25+144 \\
& =169 \\
l & =\sqrt{169}=13 \mathrm{~cm}
\end{aligned}
$$

$$
V_{01}=\frac{1}{3} \pi r^{2} \mathrm{~L}=\frac{1}{3} \pi \times s^{2} \times 12=314 \mathrm{~cm}^{3}
$$

$$
\begin{aligned}
\text { Total surface are } & =\text { curved surface area }+ \text { base } \\
& =\pi r e+\pi r^{2} \\
& =\pi \times 5 \times 13+\pi \times 5^{2} \\
& =283 \mathrm{~cm}^{3}
\end{aligned}
$$

5) A cone has Volume $1000 \mathrm{~cm}^{3}$ and a height of 10 cm . Find its radius.

$$
\begin{array}{lr}
V=\frac{1}{3} \pi r^{2} h & \\
3 V=\pi r^{2} h & r=\sqrt{\frac{3 \times 1000}{10 \pi}} \\
\frac{3 V}{\pi h}=r^{2} & r=9.77 \mathrm{~cm}
\end{array}
$$

$$
\sqrt{\frac{3 V}{\pi h}}=r
$$

Exercise

1) Find the volume and surface area of a sphere with radius
i) 6 cm
ii) $2 m$
iii) 6370 km
2) Find the volume and total surface area of a solid cone with
i) radius 7 cm , height 24 cm , slope length 25 cm
ii) radius 8 cm height 15 cm , slope length 17 cm
3) il

$$
\begin{aligned}
& V=\frac{4}{3} \pi r^{3}=\frac{4}{3} \pi \times 6^{3}=905 \mathrm{~cm}^{3} \\
& S . A=4 \pi r^{2}=4 \pi \times 6^{2}=452 \mathrm{~cm}^{2}
\end{aligned}
$$

ii)

$$
\begin{aligned}
& V=\frac{4}{3} \pi r^{3}=\frac{4}{3} \pi \times 2^{3}=33.5 \mathrm{~m}^{3} \\
& S . A=4 \pi r^{2}=4 \pi \times 2^{2}=50.3 \mathrm{~m}^{2}
\end{aligned}
$$

iii)

$$
\begin{aligned}
& V=\frac{4}{3} \pi r^{3}=\frac{4}{3} \pi \times 6370^{3}=1.08 \times 10^{12} \mathrm{~km}^{3} \\
& \text { S.A. }=4 \pi r^{2}=4 \pi \times 6370^{2}=5.09 \times 10^{8} \mathrm{~km}^{2}
\end{aligned}
$$

2) i)

$$
\begin{aligned}
& V=\frac{1}{3} \pi r^{2} L=\frac{1}{3} \pi \times 7^{2} \times 24=1232 \mathrm{cn}^{3} \\
& S . A=\pi r l+\pi r^{2}=\pi \times 7 \times 25+\pi \times 7^{2}=704 \mathrm{~cm}^{2}
\end{aligned}
$$

ii)

$$
\begin{aligned}
V & =\frac{1}{3} \pi r^{2} h=\frac{1}{3} \pi \times 8^{2} \times 15=1005 \mathrm{~cm}^{3} \\
S . A . & =\pi r l+\pi r^{2}=\pi \times 8 \times 17+\pi \times 8^{2}=628 \mathrm{~cm}^{2}
\end{aligned}
$$

