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5.

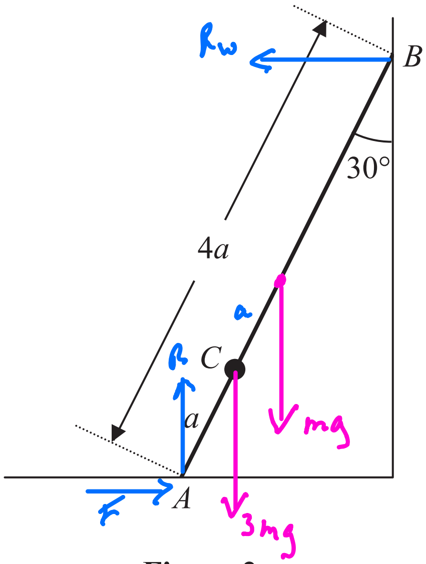

Figure 2

A ladder AB, of mass m and length $4a$, has one end A resting on rough horizontal ground. The other end B rests against a smooth vertical wall. A load of mass $3m$ is fixed on the ladder at the point C, where $AC = a$. The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as a particle. The ladder rests in limiting equilibrium making an angle of 30° with the wall, as shown in Figure 2.

Find the coefficient of friction between the ladder and the ground.

(10)

Limiting Equilibrium so $F = \mu R$

Resolve $\updownarrow$ $R = 4mg$

Moments about B

$mg \times 2a\sin 30° + 3mg \times 3a \sin 30° + F \times 4a\cos 30° = R \times 4a \sin 30°$

$mga + \frac{9}{2}mga + F \times 2\sqrt{3}a = 4mg \times 2a$

$\frac{11}{2}mga + \mu \times 4mg \times 2\sqrt{3}a = 8mga$

$\frac{11}{2} + 8\sqrt{3}\mu = 8$

$\mu = \frac{8 - \frac{11}{2}}{8\sqrt{3}}$ $\mu = 0.180$

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5.

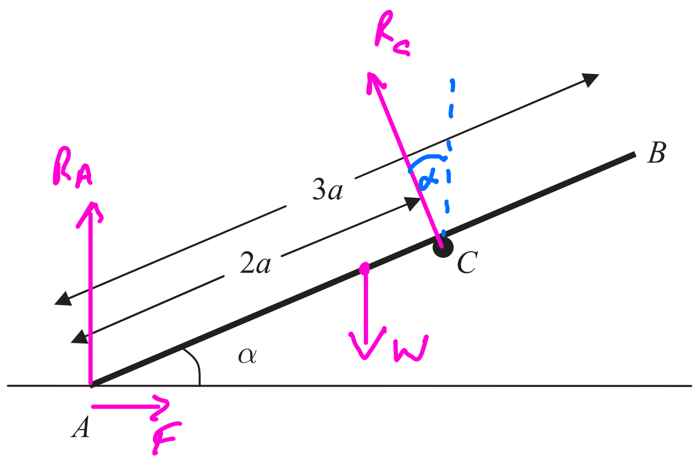


Figure 2

A plank rests in equilibrium against a fixed horizontal pole. The plank is modelled as a uniform rod AB and the pole as a smooth horizontal peg perpendicular to the vertical plane containing AB. The rod has length $3a$ and weight W and rests on the peg at C, where $AC = 2a$. The end A of the rod rests on rough horizontal ground and AB makes an angle α with the ground, as shown in Figure 2.

(a) Show that the normal reaction on the rod at A is $\frac{1}{4}(4 - 3\cos^2\alpha)W$.

(6)

Given that the rod is in limiting equilibrium and that $\cos\alpha = \frac{2}{3}$,

(b) find the coefficient of friction between the rod and the ground.

(5)

a) Resolve ↕ $R_A + R_c \cos\alpha = W$ $(*)$

Moments about A

$W \times \frac{3}{2} a \cos\alpha = R_c \times 2a$

$\frac{3}{4} W \cos\alpha = R_c$

Sub for R_c in $(*)$ $R_A + \frac{3}{4} W\cos\alpha \times \cos\alpha = W$

$R_A = W - \frac{3}{4} W\cos^2\alpha = \frac{1}{4}\left(4 - 3\cos^2\alpha\right)W$

H 2 9 4 9 8 A 0 1 2 2 4

Leave blank

Question 5 continued

Limiting equilibrium so $F = \mu R_A$

b)

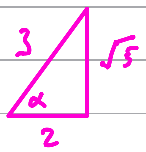
Resolve $\leftrightarrow$ $\quad F = R_c \sin\alpha$

$$F = \frac{3}{4} W \cos\alpha \sin\alpha$$

$$\mu R_A = \frac{3}{4} W \cos\alpha \sin\alpha$$

$$\mu = \frac{\frac{3}{4} W \cos\alpha \sin\alpha}{\frac{1}{4}(4 - 3\cos^2\alpha)W}$$

$$\mu = \frac{3\cos\alpha \sin\alpha}{4 - 3\cos^2\alpha}$$



$\cos\alpha = \frac{2}{3} \Rightarrow \sin\alpha = \frac{\sqrt{5}}{3}$

$$\mu = \frac{3 \times \frac{2}{3} \times \frac{\sqrt{5}}{3}}{4 - 3\left(\frac{2}{3}\right)^2} = \frac{\sqrt{5}}{4} = 0.559$$

Leave blank

2.

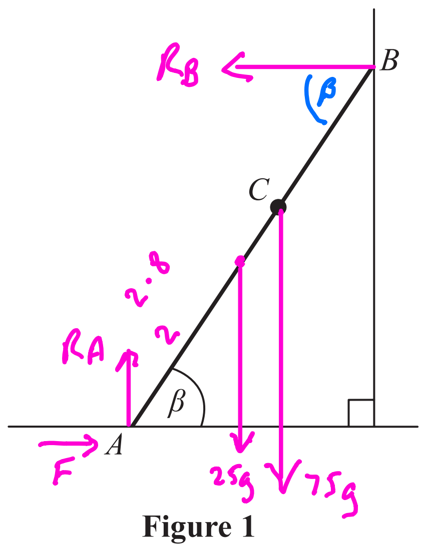


Figure 1

Figure 1 shows a ladder AB, of mass 25 kg and length 4 m, resting in equilibrium with one end A on rough horizontal ground and the other end B against a smooth vertical wall. The ladder is in a vertical plane perpendicular to the wall. The coefficient of friction between the ladder and the ground is $\frac{11}{25}$. The ladder makes an angle β with the ground. When Reece, who has mass 75 kg, stands at the point C on the ladder, where $AC = 2.8$ m, the ladder is on the point of slipping. The ladder is modelled as a uniform rod and Reece is modelled as a particle.

(a) Find the magnitude of the frictional force of the ground on the ladder. **(3)**

(b) Find, to the nearest degree, the value of β. **(6)**

(c) State how you have used the modelling assumption that Reece is a particle. **(1)**

a) Limiting Equilibrium so $F = \mu R_A$

Resolve $\updownarrow$ $R_A = 25g + 75g = 100g$

$F = \frac{11}{25} \times 100g = 44g$ N

$= 431$ N

b) Moments about A

$25g \times 2\cos\beta + 75g \times 2.8\cos\beta = R_B \times 4\sin\beta$

But resolving $\leftrightarrow$ $R_B = F = 44g$

N 3 0 0 8 3 A 0 4 2 4

Leave blank

Question 2 continued

$$260g\cos\beta = 176g\sin\beta$$

$$\frac{260g}{176g} = \frac{\sin\beta}{\cos\beta}$$

$$\frac{260}{176} = \tan\beta$$

$$\beta = 55.4^\circ$$

c) His weight acts exactly at the point C.

Leave blank

4.

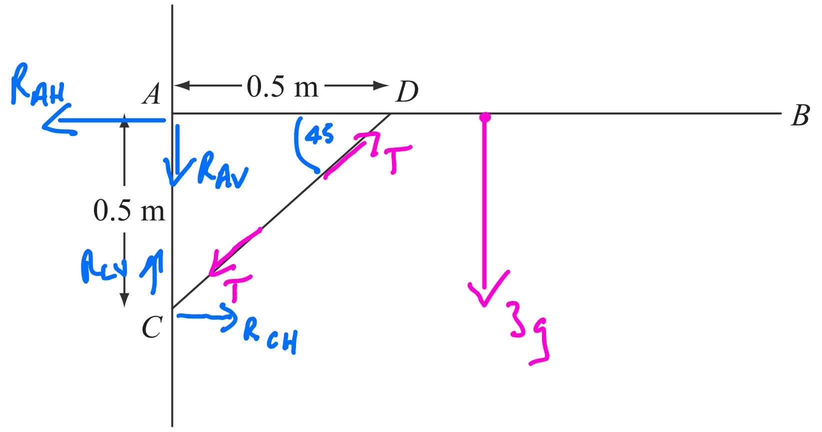


Figure 1

A uniform rod *AB*, of length 1.5 m and mass 3 kg, is smoothly hinged to a vertical wall at *A*. The rod is held in equilibrium in a horizontal position by a light strut *CD* as shown in Figure 1. The rod and the strut lie in the same vertical plane, which is perpendicular to the wall. The end *C* of the strut is freely jointed to the wall at a point 0.5 m vertically below *A*. The end *D* is freely joined to the rod so that *AD* is 0.5 m.

(a) Find the thrust in *CD*.

(4)

(b) Find the magnitude and direction of the force exerted on the rod *AB* at *A*.

(7)

a) Moments about A for rod AB

$$3g \times 0.75 = T \times 0.5 \sin 45^\circ$$

$$T = \frac{3g \times 0.75}{0.5 \sin 45^\circ} = 62.4\text{N}$$

b) Resolve $\leftrightarrow$ for rod AB

$$R_{AH} = T\cos 45 = \frac{3g \times 0.75}{0.5 \sin 45^\circ} \times \cos 45^\circ = 44.1\text{ N}$$

Resolve $\updownarrow$ for rod AB

$$T\sin 45 = 3g + R_{AV}$$

N 3 4 2 7 4 A 0 8 2 8

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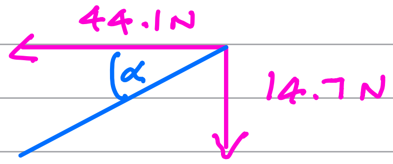
Question 4 continued

$$44.1 - 3g = R_{AV}$$

$$R_{AV} = 14.7 \text{ N}$$



$$\alpha = \tan^{-1}\left(\frac{14.7}{44.1}\right)$$

$$\alpha = 18.4^\circ$$

Magnitude of force on rod AB at A

$$= \sqrt{44.1^2 + 14.7^2} = 46.5 \text{ N}$$

Resultant force is into the wall 18.4° below the horizontal.

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6.

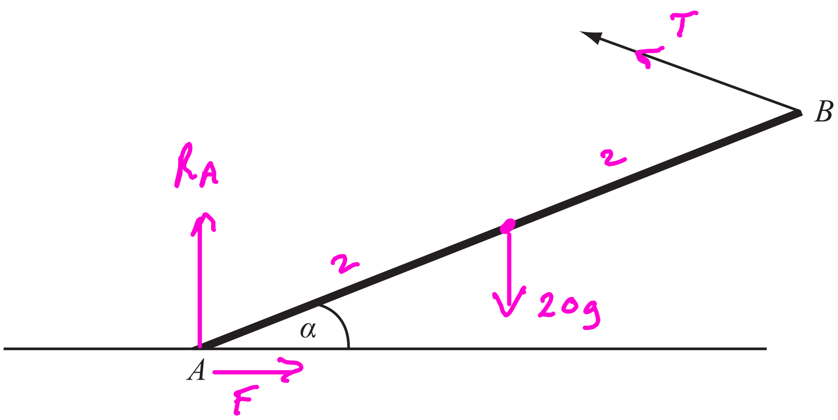

Figure 2

A uniform rod AB, of mass 20 kg and length 4 m, rests with one end A on rough horizontal ground. The rod is held in limiting equilibrium at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$, by a force acting at B, as shown in Figure 2. The line of action of this force lies in the vertical plane which contains the rod. The coefficient of friction between the ground and the rod is 0.5. Find the magnitude of the normal reaction of the ground on the rod at A.

(7)

Limiting equilibrium so $F = \mu R_A$

Moments about B

$$20g \times 2\cos\alpha + F \times 4\sin\alpha = R_A \times 4\cos\alpha$$

$$40g\cos\alpha + \mu R_A \times 4\sin\alpha = 4R_A\cos\alpha$$

$$2R_A\sin\alpha = (4R_A - 40g)\cos\alpha$$

$$2R_A\tan\alpha = 4R_A - 40g$$

$$\frac{3}{2}R_A = 4R_A - 40g$$

$$40g = \frac{5}{2}R_A$$

$$40g \times \frac{2}{5} = R_A$$

$\underline{R_A = 157\text{ N}}$

M 3 5 1 0 2 A 0 1 6 2 8

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6.

By Pythagoras

$CD = \sqrt{(3a)^2 + (2a)^2}$

$= \sqrt{13}\, a$

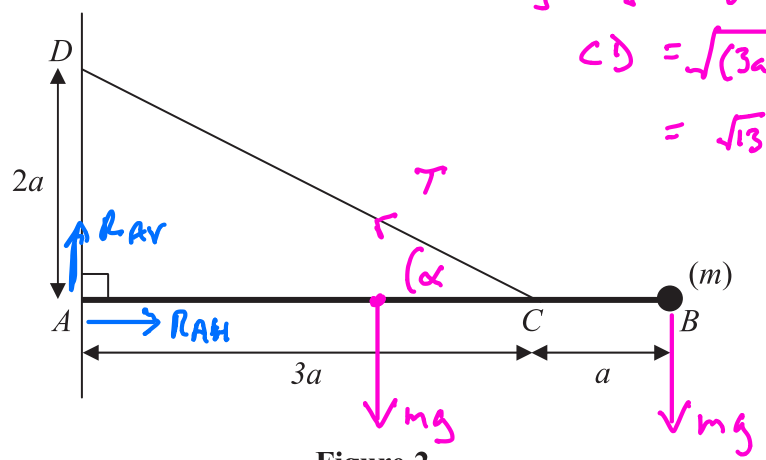

Figure 2

Figure 2 shows a uniform rod AB of mass m and length $4a$. The end A of the rod is freely hinged to a point on a vertical wall. A particle of mass m is attached to the rod at B. One end of a light inextensible string is attached to the rod at C, where $AC = 3a$. The other end of the string is attached to the wall at D, where $AD = 2a$ and D is vertically above A. The rod rests horizontally in equilibrium in a vertical plane perpendicular to the wall and the tension in the string is T.

(a) Show that $T = mg\sqrt{13}$.

(5)

The particle of mass m at B is removed from the rod and replaced by a particle of mass M which is attached to the rod at B. The string breaks if the tension exceeds $2mg\sqrt{13}$. Given that the string does not break,

(b) show that $M \leqslant \frac{5}{2}m$.

(3)

Moments about A

a)

$T \times 3a \sin\alpha = mg \times 2a + mg \times 4a$

$T \times 3a \times \frac{2a}{\sqrt{13}\, a} = 6mga$

$\frac{6}{\sqrt{13}} T a = 6mga$

$T = \sqrt{13}\, mg$

Leave blank

Question 6 continued

b) $T \times 3a\sin\alpha = mg \times 2a + Mg \times 4a$

$T \times 3a \frac{2}{\sqrt{13}} = 2ag(m + 2M)$

$T \leq 2mg\sqrt{13}$

$\therefore \quad 2mg\sqrt{13} \times \frac{6a}{\sqrt{13}} \geq 2ag(m+2M)$

$\frac{12mga}{2ag} \geq m + 2M$

$6m \geq m + 2M$

$5m \geq 2M$

$\frac{5m}{2} \geq M$

$M \leq \frac{5m}{2}$