

Negative Binomial Distribution

For successive trials with probability of success p

The number of trials required to get r successes has the negative binomial distribution

$$P(X=x) = p(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$$

$$x = r, r+1, r+2, \dots$$

Exercise 3C

7) $X \sim \text{Negative } B(r, p)$

a) $X \sim \text{Negative } B(7, 0.8)$

$$\begin{aligned} P(X=10) &= \binom{9}{6} 0.8^6 \times 0.2^3 \times 0.8 \\ &= 0.1409 \end{aligned}$$

b) constant prob and independent trials

c) $X \sim B(12, 0.8)$ $P(X \geq 7) = 1 - P(X \leq 6)$
 $= 1 - 0.0194$
 $= 0.9806$

d) $X \sim B(20, 0.8)$ $P(X \leq 14) = 0.1958$

$$q) \quad X \sim \text{Negative Binomial} \quad B(r, p)$$

$$a) \quad P(X=10) = \binom{9}{4} \times 0.7^4 \times 0.3^5 \times 0.7 \\ = \underline{\hspace{2cm}} 0.0515$$

$$b) \quad P(X \leq 6) = 1 - P(X > 6)$$

$$\begin{aligned} Y &\sim B(6, 0.7) & 1 - P(Y \leq 4) \\ P(Y \leq 4) &= 1 - 0.5798 \\ &= \underline{\hspace{2cm}} 0.4202 \end{aligned}$$

$$c) \quad P(X \leq 15) = 1 - P(X > 15)$$

$$\begin{aligned} Y &\sim B(15, 0.7) & 1 - 6.722 \times 10^{-4} \\ P(Y \leq 4) &= \underline{\hspace{2cm}} 0.9993 \end{aligned}$$

$$d) \quad P(X > 12) = P(X \leq 4)$$

$$Y \sim B(12, 0.7) = \underline{\hspace{2cm}} 0.0095$$

Negative Binomial $X \sim \text{Negative Binomial}(r, p)$

$$\text{Mean} = E(X) = \mu = \frac{r}{p}$$

$$\text{Variance} \approx \text{Var}(X) = \sigma^2 = \frac{r(1-p)}{p^2}$$

Exercise 3D

7) b) $X \sim \text{Negative } B(5, 0.7)$

$$E(X) = \mu = \frac{5}{0.7} = \frac{50}{7} = 7.143$$

$$\text{Var}(X) = \sigma^2 = \frac{5(1-0.7)}{0.7^2} = \frac{150}{49}$$

$$\text{s.d. } \sigma = \sqrt{\frac{150}{49}} = 1.750$$

a) $X \sim \text{Negative } B(3, p)$

$$E(X) = \frac{r}{p}$$

$$18.75 = \frac{3}{p} \quad p = \frac{3}{18.75} = 0.16$$

b) $\sigma^2 = \frac{r(1-p)}{p^2} = \frac{3(1-0.16)}{0.16^2} = 98.4375$

c) $X \sim \text{Neg } B(5, 0.24)$

$$E(X) = \frac{r}{p} = \frac{5}{0.24} = 20.83$$

d) $P(X > 20) = P(Y \leq 4)$

$$Y \sim B(20, 0.24) = 0.4561$$
