Jan 05

8 At a doctor's surgery, records show that 20% of patients who make an appointment fail to turn up. During afternoon surgery the doctor has time to see 16 patients.

There are 16 appointments to see the doctor one afternoon.

(i) Find the probability that all 16 patients turn up. [2]

(ii) Find the probability that more than 3 patients do not turn up. [3]

To improve efficiency, the doctor decides to make more than 16 appointments for afternoon surgery, although there will still only be enough time to see 16 patients. There must be a probability of at least 0.9 that the doctor will have enough time to see all the patients who turn up.

(iii) The doctor makes 17 appointments for afternoon surgery. Find the probability that at least one patient does not turn up. Hence show that making 17 appointments is satisfactory. [3]

(iv) Now find the greatest number of appointments the doctor can make for afternoon surgery and still have a probability of at least 0.9 of having time to see all patients who turn up. [4]

A computerised appointment system is introduced at the surgery. It is decided to test, at the 5% level, whether the proportion of patients failing to turn up for their appointments has changed. There are always 20 appointments to see the doctor at morning surgery. On a randomly chosen morning, 1 patient does not turn up.

(v) Write down suitable hypotheses and carry out the test. [7]

Let X be no shows

$X \sim B(16, 0.2)$

i) $P(X=0) = 0.8^{16} = \underline{\underline{0.0281}}$

ii) $P(X>3) = 1 - P(X \leq 3)$

$= 1 - 0.5981$

$= \underline{\underline{0.4019}}$

iii) $X \sim B(\overset{n}{17}, \overset{p}{0.2})$

$P(X \geq 1) = 1 - P(X=0)$

$= 1 - 0.8^{17}$

$= 0.9775 > 0.9$

$\therefore$ prob doc sees all patients > 0.9

so 17 appts ok

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iv) $X \sim B(18, 0.2)$ $\quad P(X \geq 2)$

$= 1 - P(X \leq 1)$

$= 1 - 0.0991$

$= 0.9009 > 0.9$

$X \sim B(19, 0.2)$ $\quad P(X \geq 3)$

$= 1 - P(X \leq 2)$

$= 1 - 0.2369$

$= 0.7631 < 0.9$

Greatest number of appts $= 18$

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v) $X \sim B(\overset{n}{20}, \overset{p}{0.2})$

$H_0: p = 0.2$

$H_1: p \neq 0.2$

where p is prob a randomly chosen patient fails to turn up

$P(X \leq 1) = 0.0692 > 0.025$ $(2\frac{1}{2}\%)$

Not significant so accept H_0

There is insufficient evidence to support the view that the probability of a no show has changed. Accept it is still 0.2

Jun 05

7 A game requires 15 identical ordinary dice to be thrown in each turn.

Assuming the dice to be fair, find the following probabilities for any given turn.

(i) No sixes are thrown. [2]

(ii) Exactly four sixes are thrown. [3]

(iii) More than three sixes are thrown. [2]

David and Esme are two players who are not convinced that the dice are fair. David believes that the dice are biased against sixes, while Esme believes the dice to be biased in favour of sixes.

In his next turn, David throws no sixes. In her next turn, Esme throws 5 sixes.

(iv) Writing down your hypotheses carefully in each case, decide whether

(*A*) David's turn provides sufficient evidence at the 10% level that the dice are biased against sixes, [5]

(*B*) Esme's turn provides sufficient evidence at the 10% level that the dice are biased in favour of sixes. [4]

(v) Comment on your conclusions from part **(iv)**. [2]

i) $X \sim B(15, \frac{1}{6})$ X = number of sixes

$P(X=0) = \left(\frac{5}{6}\right)^{15} = 0.0649$

ii) $P(X=4) = {}^{15}C_4 \times \left(\frac{1}{6}\right)^4 \times \left(\frac{5}{6}\right)^{11}$

$= 0.1418$

iii) $P(X > 3) = 1 - P(X \leq 3)$

$= 1 - 0.7685$

$= 0.2315$

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iv) A) David $\quad X$ = number of sixes

$X \sim B(\overset{n}{15}, \overset{p}{\tfrac{1}{6}})$

$H_0: p = \frac{1}{6}$ $\qquad p$ = prob of a 6

$H_1: p < \frac{1}{6}$ $\qquad E(X) = np$

$= 2.5$

$P(X \leq 0) = \left(\frac{5}{6}\right)^{15} = 0.0649 < 10\%$ $\quad$ 10% sig level

Reject H_0 and Accept H_1

There is sufficient evidence to support the view the dice are biased against sixes

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B) Esme $\quad Y \sim B(15, \frac{1}{6})$

$H_0: p = \frac{1}{6}$

$H_1: p > \frac{1}{6}$

$P(X \geq 5)$

$= 1 - P(X \leq 4)$

$= 1 - 0.9102$

$= 0.0898 < 10\%$

$\therefore$ Reject H_0 and Accept H_1

There is sufficient evidence to support the view the dice are biased in favour of 6s

v) Different conclusions reached but from 2 different samples

Homework

Jan 06

3 Over a long period of time, 20% of all bowls made by a particular manufacturer are imperfect and cannot be sold.

(i) Find the probability that fewer than 4 bowls from a random sample of 10 made by the manufacturer are imperfect. [2]

The manufacturer introduces a new process for producing bowls. To test whether there has been an improvement, each of a random sample of 20 bowls made by the new process is examined. From this sample, 2 bowls are found to be imperfect.

(ii) Show that this does not provide evidence, at the 5% level of significance, of a reduction in the proportion of imperfect bowls. You should show your hypotheses and calculations clearly. [6]

Jun 06

7 A geologist splits rocks to look for fossils. On average 10% of the rocks selected from a particular area do in fact contain fossils.

The geologist selects a random sample of 20 rocks from this area.

(i) Find the probability that

(A) exactly one of the rocks contains fossils, [3]

(B) at least one of the rocks contains fossils. [3]

(ii) A random sample of n rocks is selected from this area. The geologist wants to have a probability of 0.8 or greater of finding fossils in at least one of the n rocks. Find the least possible value of n. [3]

(iii) The geologist explores a new area in which it is claimed that less than 10% of rocks contain fossils. In order to investigate the claim, a random sample of 30 rocks from this area is selected, and the number which contain fossils is recorded. A hypothesis test is carried out at the 5% level.

(A) Write down suitable hypotheses for the test. [3]

(B) Show that the critical region consists only of the value 0. [4]

(C) In fact, 2 of the 30 rocks in the sample contain fossils. Complete the test, stating your conclusions clearly. [2]

Jan 07

7 When onion seeds are sown outdoors, on average two-thirds of them germinate. A gardener sows seeds in pairs, in the hope that at least one will germinate.

(i) Assuming that germination of one of the seeds in a pair is independent of germination of the other seed, find the probability that, if a pair of seeds is selected at random,

(A) both seeds germinate,

(B) just one seed germinates,

(C) neither seed germinates. [3]

(ii) Explain why the assumption of independence is necessary in order to calculate the above probabilities. Comment on whether the assumption is likely to be valid. [2]

(iii) A pair of seeds is sown. Find the expectation and variance of the number of seeds in the pair which germinate. [3]

(iv) The gardener plants 200 pairs of seeds. If both seeds in a pair germinate, the gardener destroys one of the two plants so that only one is left to grow. Of the plants that remain after this, only 85% successfully grow to form an onion. Find the expected number of onions grown from the 200 pairs of seeds. [3]

If the seeds are sown in a greenhouse, the germination rate is higher. The seed manufacturing company claims that the germination rate is 90%. The gardener suspects that the rate will not be as high as this, and carries out a trial to investigate. 18 randomly selected seeds are sown in the greenhouse and it is found that 14 germinate.

(v) Write down suitable hypotheses and carry out a test at the 5% level to determine whether there is any evidence to support the gardener's suspicions. [7]