General Motion

Acceleration Variable

3. A particle P of mass 0.3 kg is moving under the action of a single force **F** newtons. At time t seconds the velocity of P, **v** m s^{-1}, is given by

$$\mathbf{v} = 3t^2\mathbf{i} + (6t - 4)\mathbf{j}.$$

(a) Calculate, to 3 significant figures, the magnitude of **F** when $t = 2$.

(5)

When $t = 0$, P is at the point A. The position vector of A with respect to a fixed origin O is $(3\mathbf{i} - 4\mathbf{j})$ m. When $t = 4$, P is at the point B.

(b) Find the position vector of B.

(5)

$\underline{v} = \begin{pmatrix} 3t^2 \\ 6t-4 \end{pmatrix}$ $\quad \underline{a} = \frac{d\underline{v}}{dt} = \begin{pmatrix} 6t \\ 6 \end{pmatrix}$

$\underline{F} = m\underline{a} = 0.3\begin{pmatrix} 6t \\ 6 \end{pmatrix}$

When $t = 2$ $\quad \underline{F} = 0.3\begin{pmatrix} 12 \\ 6 \end{pmatrix} = \begin{pmatrix} 3.6 \\ 1.8 \end{pmatrix}$

$$|\underline{F}| = \sqrt{3.6^2 + 1.8^2} = 4.02\ \text{N}$$

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$$\underline{r} = \int \underline{v}\, dt = \int \begin{pmatrix} 3t^2 \\ 6t-4 \end{pmatrix} dt$$

$$\underline{r} = \begin{pmatrix} t^3 + c_1 \\ 3t^2 - 4t + c_2 \end{pmatrix}$$

$t = 0$

$$\underline{r} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \Rightarrow c_1 = 3,\ c_2 = -4$$

$$\underline{r} = \begin{pmatrix} t^3 + 3 \\ 3t^2 - 4t - 4 \end{pmatrix}$$

when $t = 4$ $\quad \underline{r} = \begin{pmatrix} 4^3 + 3 \\ 3(4)^2 - 4(4) - 4 \end{pmatrix} = \begin{pmatrix} 67 \\ 28 \end{pmatrix}$

Position vector of B = $67\underline{i} + 28\underline{j}$