Homework Review

Ex 6c Q7

$P(2,2)$

$Q(2+\sqrt{3}, 5)$

$R(2-\sqrt{3}, 5)$

$$(x-2)^2 + (y-4)^2 = r^2$$

$$(2-2)^2 + (2-4)^2 = r^2$$

$$0 + 4 = r^2$$

$$\underline{r = 2}$$

$$|PQ| = \sqrt{(2+\sqrt{3}-2)^2 + (5-2)^2} = \sqrt{3+9} = \sqrt{12}$$

$$|PR| = \sqrt{(2-\sqrt{3}-2)^2 + (5-2)^2} = \sqrt{3+9} = \sqrt{12}$$

$$|QR| = \sqrt{(2\sqrt{3})^2 + 0} = \sqrt{12}$$

Equilateral since all sides $= \sqrt{12}$

Aside

Consider $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b^2 - 4ac$ is called the discriminant

If $b^2 - 4ac > 0$ two real roots

If $b^2 - 4ac = 0$ one repeated root

If $b^2 - 4ac < 0$ no real roots

Exercise 6D

7) $y = 2x - 2$ (1) $\qquad (x-2)^2 + (y-2)^2 = 20$ (2)

Sub for y in (2)

$$(x-2)^2 + (2x-2-2)^2 = 20$$

$$x^2 - 4x + 4 + (2x-4)^2 = 20$$

$$x^2 - 4x + 4 + 4x^2 - 16x + 16 = 20$$

$$5x^2 - 20x + 20 = 20$$

$$5x^2 - 20x = 0$$

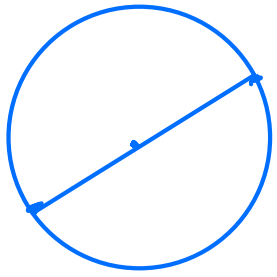
$$5x(x-4) = 0$$

Either $x = 0$ or $x = 4$

$y = 2(0) - 2$ $\qquad$ $y = 2(4) - 2$

$y = -2$ $\qquad$ $y = 6$

$A(0, -2)$ $\qquad$ $B(4, 6)$



$$|AB| = \sqrt{(4-0)^2 + (6--2)^2}$$

$$= \sqrt{16 + 64} = \sqrt{80}$$

$$= \sqrt{16 \times 5} = 4\sqrt{5}$$

Radius $= \sqrt{20} \approx 2\sqrt{5}$

$455 = 2 \times 255$ $\therefore$ AB is a diameter

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9) $(x-4)^2 + (y+7)^2 = 50$

$x - y - 5 = 0$

$x = y + 5$

Sub for x in circle

$(y+5-4)^2 + (y+7)^2 = 50$

$(y+1)^2 + (y+7)^2 = 50$

$y^2 + 2y + 1 + y^2 + 14y + 49 = 50$

$2y^2 + 16y = 0$

$2y(y+8) = 0$

$y = 0$ or $y = -8$

$x = 0 + 5$ $\quad$ $x = -8 + 5$

$x = 5$ $\quad$ $x = -3$

$A(5, 0)$ $\quad$ $B(-3, -8)$

gradient AB $= \dfrac{-8-0}{-3-5} = \dfrac{-8}{-8} = 1$

$\perp$ bisector gradient $= -1$

AB midpoint $\left(\dfrac{5+-3}{2}, \dfrac{0+-8}{2}\right) = (1, -4)$

$$y - y_1 = m(x - x_1)$$

$$y - -4 = -1(x - 1)$$

$$y + 4 = -x + 1$$

$$y = -x - 3$$

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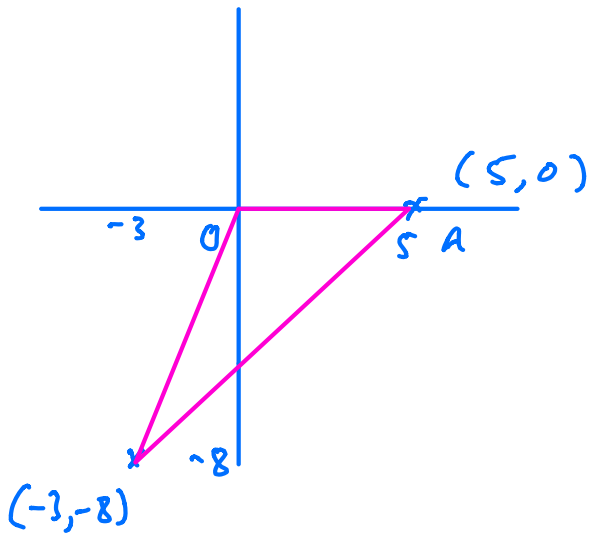
c) Centre of circle $= (4, -7)$

Show $(4, -7)$ on $\perp$ bisector

$$-7 = -4 - 3 \quad \checkmark$$



Area $\triangle AOB$

$= \frac{1}{2}$ base $\times$ height

$= \frac{1}{2} \times 5 \times 8$

$= 20$ units^2

Exercise 6E

5) $x^2 + 18x + y^2 - 2y + 29 = 0$

a) $(-7, -6)$

$$(-7)^2 + 18(-7) + (-6)^2 - 2(-6) + 29$$

$$= 49 - 126 + 36 + 12 + 29$$

$$= 0 \quad \checkmark \qquad \text{on circle}$$

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b) $$(x+9)^2 - 81 + (y-1)^2 - 1 + 29 = 0$$

$$(x+9)^2 + (y-1)^2 = 53$$

Centre $A(-9, 1)$

$P(-7, -6)$

gradient of radius $AP = \dfrac{1 - -6}{-9 - -7} \quad \dfrac{7}{-2} = -\dfrac{7}{2}$

gradient of tgt at $P = +\dfrac{2}{7}$

$$y - y_1 = m(x - x_1)$$

$$y - -6 = \frac{2}{7}(x - -7)$$

$$y + 6 = \frac{2}{7}(x + 7)$$

$$y = \frac{2}{7}x + 2 - 6$$

$$y = \frac{2}{7}x - 4$$

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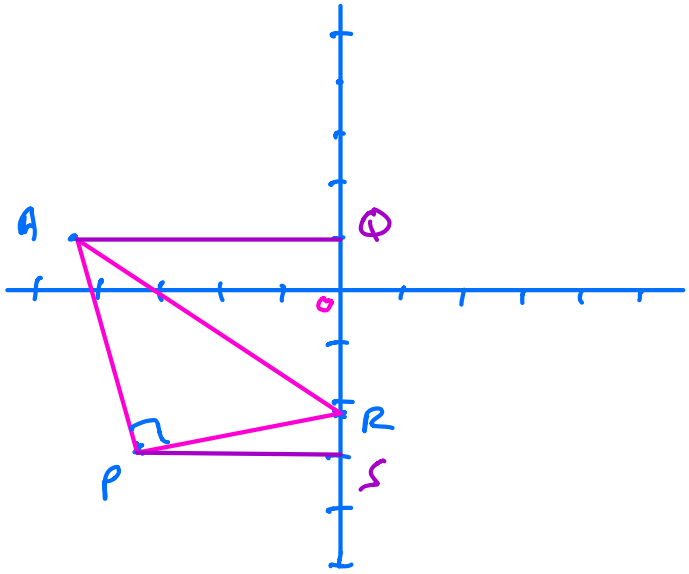
c) On y-axis $x = 0$

$y = 0 - 4 \qquad R(0, -4)$

$A(-9, 1)$

$P(-7, -6)$

$R(0, -4)$



$\triangle APR$ = Area Trap $AQSP - \triangle AQR - \triangle PRS$

$= \frac{1}{2}(9+7)7 - \frac{1}{2} \times 5 \times 4 - \frac{1}{2} \times 2 \times 7$

$= 56 - 22\frac{1}{2} - 7$

$= 26.5$

Hwk Ex 6D 6, 8, 10, 12
