

Functions Questions

Q1.

f and g are functions such that

$$f(x) = 3x^2 \quad \text{and} \quad g(x) = \frac{1}{x-2}$$

Find $gf(4)$.

Give your answer as a fraction.

$$f(4) = 3(4)^2 = 48$$

$$gf(4) = g(48) = \frac{1}{48-2}$$

$$\frac{1}{46}$$

(Total for question = 2 marks)

Q2.

f and g are functions such that

$$f(x) = \frac{2}{x^2} \quad \text{and} \quad g(x) = 4x^3$$

(a) Find $f(-5)$

$$f(-5) = \frac{2}{(-5)^2} = \frac{2}{25}$$

$$\frac{2}{25}$$

(1)

(b) Find $fg(1)$

$$g(1) = 4(1)^3 = 4$$

$$fg(1) = f(4) = \frac{2}{4^2}$$

$$= \frac{1}{8}$$

(2)

(Total for question = 3 marks)

Q3.

The function f is such that

$$f(x) = 4x - 1$$

(a) Find $f^{-1}(x)$

$$\begin{aligned} \text{Let } y &= 4x - 1 \\ \text{Swap } x &= 4y - 1 \\ x + 1 &= 4y \\ \frac{x + 1}{4} &= y \end{aligned}$$

$$f^{-1}(x) = \frac{x + 1}{4} \quad (2)$$

The function g is such that

$$g(x) = kx^2 \text{ where } k \text{ is a constant.}$$

Given that $fg(2) = 12$

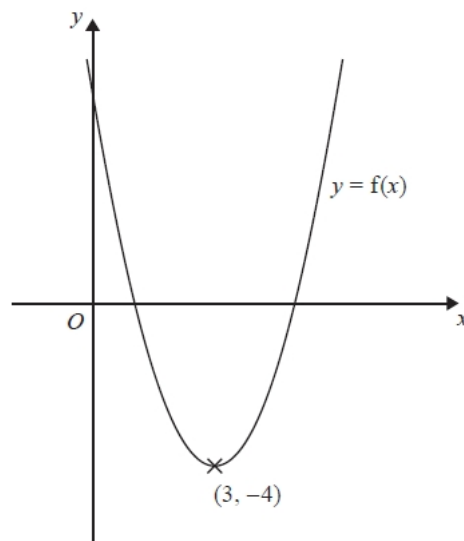
(b) work out the value of k

$$\begin{aligned} 16k - 1 &= 12 \\ 16k &= 13 \\ k &= \frac{13}{16} \end{aligned}$$

$$\begin{aligned} g(2) &= k(2)^2 = 4k \\ fg(2) &= f(4k) \\ &= 4(4k) - 1 \\ &= 16k - 1 \end{aligned}$$
$$k = \frac{13}{16} \quad (2)$$

(Total for question = 4 marks)

Q4.



The diagram shows part of the curve with equation $y = f(x)$.

The coordinates of the minimum point of this curve are (3, -4)

Write down the coordinates of the minimum point of the curve with equation

(i) $y = f(x) + 3$

(..... 3, -1)

(ii) $y = f(2x)$

(..... 1.5, -4)

(iii) $y = f(-x)$

(..... -3, -4)

(Total for Question is 3 marks)

Q5.

For all values of x

$$f(x) = 2x - 3 \quad \text{and} \quad g(x) = x^2 + 2$$

(a) Find $g(-4)$

$$g(-4) = (-4)^2 + 2$$

$$= 18$$

.....

(1)

(b) Show that $gf(x) = 4x^2 - 12x + 11$

$$\begin{aligned} gf(x) &= g(2x-3) = (2x-3)^2 + 2 \\ &= 4x^2 - 12x + 9 + 2 \\ &= 4x^2 - 12x + 11 \end{aligned}$$

(2)

(c) Solve $fg(x) = gf(x)$

$$\begin{aligned} fg(x) &= f(x^2+2) \\ &= 2(x^2+2) - 3 \\ &= 2x^2 + 4 - 3 \\ &= 2x^2 + 1 \end{aligned}$$

$$\begin{aligned} 4x^2 - 12x + 11 &= 2x^2 + 1 \\ 2x^2 - 12x + 10 &= 0 \end{aligned}$$

$$\begin{aligned} x^2 - 6x + 5 &= 0 \\ (x-1)(x-5) &= 0 \end{aligned}$$

(4)

(Total for question = 7 marks)

$$\underline{x = 1} \quad \text{or} \quad \underline{x = 5}$$

Q6.

$$f(x) = \frac{1}{x+2} + \frac{1}{x-3}$$

(a) Work out $f(5)$

Give your answer as a fraction.

$$\begin{aligned} f(5) &= \frac{1}{5+2} + \frac{1}{5-3} \\ &= \frac{1}{7} + \frac{1}{2} \\ &= \frac{2+7}{14} \end{aligned}$$

$$= \frac{9}{14}$$

(2)

(b) Write down a value of x for which $f(x)$ is not defined.

$$x = -2 \text{ or } x = 3$$

(1)

Given that $f(x) = 4$

(c) find the possible values of x .

Give your answer in the form $\frac{p \pm \sqrt{q}}{r}$ where p , q and r are positive integers.

$$\frac{1}{(x+2)} + \frac{1}{(x-3)} = 4$$

$$x-3 + x+2 = 4(x+2)(x-3)$$

$$2x-1 = 4(x^2-x-6)$$

$$2x-1 = 4x^2-4x-24$$

$$0 = 4x^2-6x-23$$

$$x = \frac{6 \pm \sqrt{36+4 \times 4 \times 23}}{8}$$

$$x = \frac{6 \pm \sqrt{404}}{8}$$

(5)

(Total for question = 8 marks)

Q7.

$$f(x) = x^3$$

$$g(x) = 4x - 1$$

(a) Find $fg(2)$

$$g(2) = 4(2) - 1 = 7$$

$$fg(2) = f(7) = 7^3$$

$$= 343$$

(2)

$$h(x) = fg(x)$$

(b) Find an expression for $h^{-1}(x)$

$$h(x) = fg(x) = f(4x-1) = (4x-1)^3$$

$$\text{Let } y = (4x-1)^3$$

$$\text{Swap } x = (4y-1)^3$$

$$\sqrt[3]{x} = 4y-1$$

$$\sqrt[3]{x} + 1 = 4y$$

$$\frac{\sqrt[3]{x} + 1}{4} = y$$

$$h^{-1}(x) = \frac{\sqrt[3]{x} + 1}{4}$$

(3)

(Total for question = 5 marks)

Q8.

The functions f and g are such that

$$f(x) = 5x + 3$$

$$g(x) = ax + b$$

where a and b are constants.

$$g(3) = 20 \quad \text{and} \quad f^{-1}(33) = g(1)$$

Find the value of a and the value of b .

$$\text{Let } y = 5x + 3$$

$$\text{Swap } x = 5y + 3$$

$$x - 3 = 5y$$

$$\frac{x-3}{5} = y$$

$$f^{-1}(x) = \frac{x-3}{5}$$

$$f^{-1}(33) = \frac{33-3}{5} = \frac{30}{5} = 6$$

$$g(1) = a + b$$

$$\therefore \underline{a + b = 6}$$

$$\underline{g(3) = 20 = 3a + b}$$

$$a + b = 6 \quad \textcircled{1}$$

$$3a + b = 20 \quad \textcircled{2}$$

$$\textcircled{2} - \textcircled{1} \quad 2a = 14$$

$$\underline{a = 7}$$

$$7 + b = 6$$

$$b = 6 - 7$$

$$\underline{b = -1}$$

$$a = \underline{7}$$

$$b = \underline{-1}$$

(Total for question = 5 marks)

Q9.

The functions f and g are such that

$$f(x) = 3(x - 4) \text{ and } g(x) = \frac{x}{5} + 1$$

(a) Find the value of $f(10)$

$$\begin{aligned} f(10) &= 3(10 - 4) \\ &= 3 \times 6 \end{aligned}$$

$$= 18$$

(1)

(b) Find $g^{-1}(x)$

$$\text{Let } y = \frac{x}{5} + 1$$

$$\text{Swap } x = \frac{y}{5} + 1$$

$$x - 1 = \frac{y}{5}$$

$$5(x - 1) = y$$

$$g^{-1}(x) = 5(x - 1)$$

(2)

(c) Show that $ff(x) = 9x - 48$

$$\begin{aligned} ff(x) &= f(3(x - 4)) = f(3x - 12) \\ &= 3(3x - 12 - 4) \\ &= 3(3x - 16) \\ &= 9x - 48 \end{aligned}$$

(2)

(Total for question = 5 marks)

Q10.

$$f(x) = 3x^2 - 2x - 8$$

Express $f(x + 2)$ in the form $ax^2 + bx$

$$f(x + 2) = 3(x + 2)^2 - 2(x + 2) - 8$$

$$= 3(x^2 + 4x + 4) - 2x - 4 - 8$$

$$= 3x^2 + 12x + 12 - 2x - 4 - 8$$

$$= 3x^2 + 10x$$

$$= 3x^2 + 10x$$

(Total for question is 3 marks)