Solutions

Exponentials Mastermind

1. The population of rabbits increases exponentially. In year 1 there were 400 rabbits. By year 3 there were 529 rabbits. How many rabbits were there by year 5?

Yr 1 $\quad 400$

Yr 3 $\quad 400r^2 = 529$

$$r^2 = \frac{529}{400}$$

$$r = \sqrt{\frac{529}{400}} = \frac{23}{20}$$

Yr 5 $\quad 400r^4 = 400 \times \left(\frac{23}{20}\right)^4 = 699.6$

≈ 700 rabbits

2. A bank account accrues i% interest each year. In year 1, the bank account had £5000 in it. In year 4, the bank account had £5306.04 in it. How much was in the account in year 10?

Yr 1 $\quad £5000$

Yr 4 $\quad £5000 \times r^3 = £5306.04$

$$r^3 = \frac{5306.4}{5000}$$

$$r = \sqrt[3]{\frac{5306.04}{5000}} = 1.02$$

so $i\% = 2\%$

Yr 10 $\quad £5000 \times r^9$

$= £5000 \times 1.02^9 = £5975.46$

3. The bacteria in a petri dish after one hour is 10 per cm^2. After 5 hours it is 1690 per cm^2. How many bacteria (per cm^2) will there be in the dish after 9 hours?

1 hr $\quad 10$ per cm^2

5 hrs $\quad 10r^4 = 1690$

$$r^4 = \frac{1690}{10}$$

$$r = \sqrt[4]{169} = \sqrt[2]{13}$$

9 hrs $\quad 10r^8 = 10 \times 13^4 = 285,610$ per cm^2

4. In 2010 in Tidmouth, 40,000 people were unemployed. By 2012 this had decreased to 39,601. Assuming the unemployment rate is decreasing exponentially, how many people would you expect to be unemployed in Tidmouth by 2020?

2010 $\quad 40000$

2012 $\quad 40000r^2 = 39601$

$$r^2 = \frac{39601}{40000}$$

$$r = \sqrt{\frac{39601}{40000}} = 0.995$$

2020 $\quad 40000 \times 0.995^{10} = 38044$

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5. In 2008, 1.7m iPhones were sold. In 2012, 35.2m iPhones were sold. Assuming the sales of iPhones increases exponentially, how many iPhones should be expected to be sold in 2017?

2008 1.7 m

2012 $1.7 \times r^4 = 35.2$

$r^4 = \frac{35.2}{1.7}$

$r = \sqrt[4]{\frac{35.2}{1.7}} = 2.13316002$

2017 $1.7 \times 2.13316002^9 = 1554.7$ m

6. A newly born octopus weighs 180g. After 2 days it weighed 202.248g. The octopus' weight increases exponentially for the first few weeks of its life. How much will the octopus weigh after 2 weeks?

0 days 180g

2 days $180 \times r^2 = 202.248$

$r^2 = \frac{202.248}{180}$

$r = \sqrt{\frac{202.248}{180}} = 1.06$

14 days $180 \times 1.06^{14} = 406.963$ g

7. A vase contains 800ml of water. A daffodil is placed in the vase and after 5 days the vase contains 619ml of water. Assuming the water in the vase decreases exponentially, how much water was in the vase after 8 days?

0 days $\quad 800\text{ ml}$

5 days $\quad 800r^5 = 619$

$$r^5 = \frac{619}{800}$$

$$r = \sqrt[5]{\frac{619}{800}} = 0.94499$$

8 days $\quad 800 \times 0.94499^8 = 530.69\text{ ml}$

$$\approx 531\text{ ml}$$

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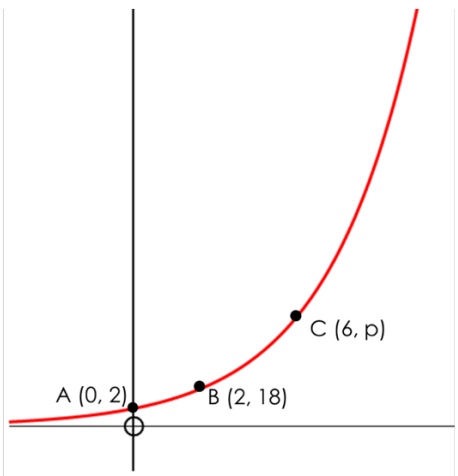
8. The diagram on the right shows the curve of $y = ab^x$

It passes through the points A (0, 2), B (2, 18) and C (6, p)

Find the value of p.



A(0,2) $\quad 2 = ab^0$

$$2 = a$$

B(2,18) $\quad 18 = 2 \times b^2$

$$\frac{18}{2} = b^2$$

$$9 = b^2$$

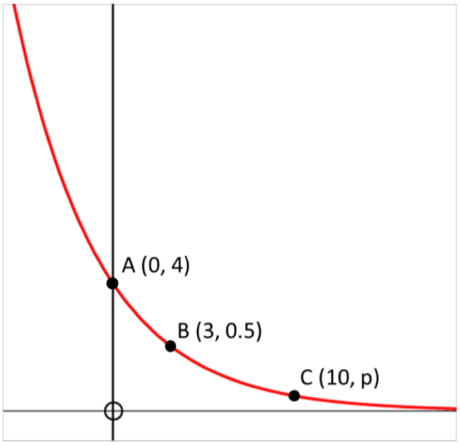
$$3 = b$$

$$y = 2 \times 3^x$$

C(6,p)

$$p = 2 \times 3^6$$

$$p = 1458$$



9. The diagram on the right shows the curve of $y = ab^x$

It passes through the points A (0, 4), B (3, 0.5) and C (10, p)

Find the value of p.

A(0,4) $4 = ab^0$

$4 = a$

B(3,0.5) $0.5 = 4 \times b^3$

$\frac{0.5}{4} = b^3$

$\frac{1}{8} = b^3$

$\frac{1}{2} = b$

$y = 4 \times 0.5^x$

C(10,p)

$p = 4 \times 0.5^{10}$

$p = \frac{1}{256}$

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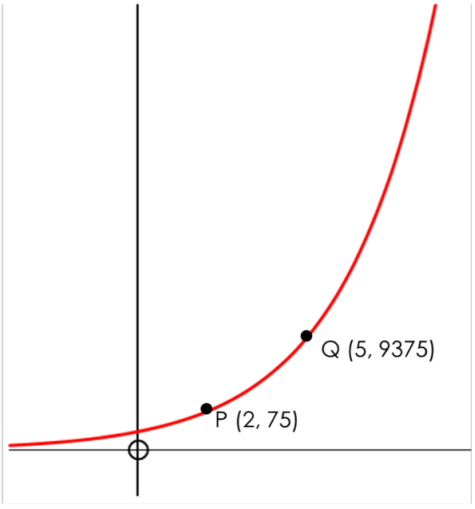
10. The diagram on the right shows the curve of $y = ab^x$

It passes through the points P (2, 75), and Q (5, 9375)

Find the values of a and b.



P(2,75) $75 = a \times b^2$ ①

Q(5,9375) $9375 = a \times b^5$ ②

② ÷ ① $\frac{9375}{75} = \frac{ab^5}{ab^2}$

$125 = b^3$

$5 = b$

Sub for b in ①

$75 = a \times 5^2$

$75 = 25a$

$\frac{75}{25} = a$

$3 = a$

$a = 3, \quad b = 5$